

Midterm #1

MATH 436/565 — Numerical Analysis

Monday, Sept 29, 2025

Please print your name:

No notes, fancy calculators or tools of any kind are permitted. There are 28 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (5 points) Suppose we wish to approximate the function $f(x) = 3 - 4x \ln(x)$.

- (a) What is the 2nd Taylor polynomial $p_2(x)$ of $f(x)$ at $x = 1$?
- (b) Provide an upper bound for the error of approximating $f(x)$ by $p_2(x)$ on the interval $[1, 2]$.

Problem 2. (2 points) Newton's method applied to $x^3 - 7$ is equivalent to fixed-point iteration of which function?

Problem 3. (5 points) Determine all fixed points of $f(x) = \frac{4}{x+3}$. For each fixed point x^* determine whether fixed-point iteration of $f(x)$ converges locally to x^* . If so, determine the exact order of convergence as well as the rate.

Problem 4. (2 points) We have learned about the Newton method, the bisection method, the regula falsi method, and the secant method. List those methods that are guaranteed to converge.

Problem 5. (2 points) Suppose that x^* is a root of $f(x)$. When does Newton's method fail to locally converge to x^* with order of convergence at least 2?

Problem 6. (2 points) Give one advantage of the bisection method over the Newton method, as well as one advantage of the Newton method over the bisection method.

advantage of the bisection method:
advantage of the Newton method:

Problem 7. (3 points) Represent -6.25 as a single precision floating-point number according to IEEE 754.

-6.25 as a single precision float:

Problem 8. (3 points) Express $15/7$ in base 2. If necessary, indicate which digits repeat.

--

Problem 9. (2 points) Express -5 in binary using the two's complement representation with 6 bits.

--

Problem 10. (2 points) Indicate two reasons why floating-point numbers are used rather than fixed-point numbers.

--

(extra scratch paper)