Midterm #1

MATH 436/565 — Numerical Analysis
Wednesday, Oct 4, 2023

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 35 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (5 points) Determine all fixed-points of $f(x) = \frac{4}{x+3}$. For each fixed-point x^* determine whether fixed-point iteration of $f(x)$ converges locally to x^*. If so, determine the exact order of convergence as well as the rate.

Problem 2. (2 points) We have learned about the Newton method, the bisection method, the regula falsi method, and the secant method. List those methods that are guaranteed to converge.

Problem 3. (2 points) Describe briefly how the regula falsi method proceeds different from the bisection method.

Armin Straub
straub@southalabama.edu

Problem 4. (3 points) Express $12/5$ in base 2. If necessary, indicate which digits repeat.

Problem 5. (2 points) Express -17 in binary using the two's complement representation with 6 bits.

Problem 6. (5 points) Consider $f(x) = (x+r)(x-2)$ where r is some constant. Suppose we want to use Newton's method to calculate the root $x^* = 2$.

(a) For which values of r is Newton's method guaranteed to converge (at least) quadratically to $x^* = 2$?

(b) Analyze the case in which Newton's method does not converge quadratically to $x^* = 2$. Does it still converge? If so, determine the order and rate of convergence.

Problem 7. (2 points)

(a) Indicate one advantage of the bisection method over the Newton method.

(b) Indicate one advantage of the Newton method over the bisection method.

Problem 8. (2 points) Newton's method applied to $x^4 - 2$ is equivalent to fixed-point iteration of which function?

Problem 9. (4 points) We wish to compute the root $\sqrt{1/2}$ of $f(x) = 2x^2 - 1$ using the bisection method.

(a) Starting with the interval $[0, 1]$, apply two iterations of bisection. What is the resulting approximation of $\sqrt{1/2}$?

(b) After how many iterations can we guarantee that the absolute error is less than 0.001?

Problem 10. (5 points) Suppose we wish to approximate the function $f(x) = 5 + 3x\ln(x)$.

(a) What is the 2nd Taylor polynomial $p_2(x)$ of $f(x)$ at $x = 1$?

(b) Provide an upper bound for the error of approximating $f(x)$ by $p_2(x)$ on the interval $[1, 2]$.

Problem 11. (3 points) Represent -4.5 as a single precision floating-point number according to IEEE 754.

-4.5 as a single precision float:

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(extra scratch paper)