Midterm #1

MATH 436 — Numerical Analysis
Thursday, Sept 29

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 36 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (6 points) Consider $f(x) = (x + r)(x^2 - 1)$ where r is some constant. Suppose we want to use Newton's method to calculate the root $x^* = 1$.

(a) For which values of r is Newton's method guaranteed to converge (at least) quadratically to $x^* = 1$?

(b) For which values of r does Newton's method converge to $x^* = 1$ faster than quadratically?

Problem 2. (3 points)

(a) Give one advantage of the secant method over the regula falsi method.

(b) Give one advantage of the regula falsi method over the secant method.

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Problem 3. (6 points) Determine all fixed-points of $f(x) = \frac{x}{x+2}$. For each fixed-point x^* determine whether fixed-point iteration of $f(x)$ converges locally to x^*. If so, determine the exact order of convergence as well as the rate.

Problem 4. (3 points) Express $19/6$ in base 2. If necessary, indicate which digits repeat.

$19/6$ in base 2:

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Problem 5. (8 points) We wish to compute the root $\sqrt{3}$ of $f(x) = x^2 - 3$ using the bisection method.

(a) Starting with the interval $[1, 2]$, apply two iterations of bisection. What is the resulting approximation of $\sqrt{3}$?

(b) After how many iterations can we guarantee that the absolute error is less than 0.001?

(c) Describe in a few words how the regula falsi method proceeds different from the bisection method.

(d) Newton's method applied to $x^2 - 3$ is equivalent to fixed-point iteration of which function $g(x)$?

Problem 6. (2 points)

(a) Suppose we use the regula falsi method to compute the root of a function $f(x)$. Several iterations result in the intervals $[1, 2], \left[\frac{7}{6}, 2\right], \left[\frac{44}{37}, 2\right], \left[\frac{273}{229}, 2\right]$.

Based on these, our approximation of the root is ____________.

(b) Express -18 in binary using the two's complement representation with 6 bits.

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Problem 7. (5 points) Suppose we wish to approximate the function $f(x) = 2x\ln(x)$.

(a) What is the 2nd Taylor polynomial $p_2(x)$ of $f(x)$ at $x = 1$?

(b) Provide an upper bound for the error of approximating $f(x)$ by $p_2(x)$ on the interval $[1, 2]$.

Problem 8. (3 points) Represent -2.5 as a single precision floating-point number according to IEEE 754.

-2.5 as a single precision float:

(extra scratch paper)