

Final: practice

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches). Any typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Problem 1. The final exam will be comprehensive, that is, it will cover the material of the whole semester.

- (a) Do the practice problems for both midterms.
- (b) Retake both midterm exams.
- (c) Do the problems below. (Solutions are posted.)

Problem 2. Use the trapezoidal rule to approximate $\int_0^1 \frac{1}{x^2+1} dx = \frac{\pi}{4}$.

- (a) Use $h = \frac{1}{3}$ and $h = \frac{1}{6}$.
- (b) Using Richardson extrapolation, combine the previous two approximations to obtain an approximation of higher order. What are absolute and relative error?
- (c) The extrapolated approximation is equivalent to the outcome of which method applied with $h = \frac{1}{6}$?

Problem 3. Consider the initial value problem $y' = xy^2 + 1$, $y(1) = 0$.

- (a) Approximate the solution $y(x)$ for $x \in [1, 3]$ using Euler's method with 3 steps. In particular, what is the approximation for $y(3)$?
- (b) What is the order of the local truncation error? The global error?
- (c) Spell out the Taylor method of order 3 for numerically solving this initial value problem.