Continued fractions

continued fraction $a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\cdots}}}$

- written as $[a_0; a_1, a_2, \ldots]$
- called simple if all a_i integers and $a_1, a_2, \ldots > 0$
- the **convergents** C_k are
  $C_0 = a_0 \quad C_1 = [a_0; a_1] \quad C_2 = [a_0; a_1, a_2] \quad \ldots$

EG $\underset{}{\overset{C_1}{[2;3]}} = 2 + \frac{1}{3} = \frac{7}{3}$ ≈ 2.333 $\quad \overset{C_2}{[2;3,4]} = 2 + \cfrac{1}{3+\frac{1}{4}}$

$= 2 + \frac{4}{13} = \frac{30}{13} \approx 2.308$

$\underset{C_3}{[2;3,4,5]} = 2 + \cfrac{1}{3 + \cfrac{1}{4 + \frac{1}{5}}} = \frac{157}{68}$ ≈ 2.309

THM The convergents C_k of $[a_0; a_1, a_2, \ldots]$ simple CF! $= x$

always converge to a value x :

$C_0 < C_2 < C_4 \; \ldots \; x \; \ldots < C_5 < C_3 < C_1$

$a_0 \qquad\qquad a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cdots}} \qquad\qquad a_0 + \frac{1}{a_1}$

$>0 \qquad >0$

EG $\frac{5}{3} = 1 + \frac{2}{3} = 1 + \cfrac{1}{3/2} = 1 + \cfrac{1}{1 + \frac{1}{2}}$

repeat! $1 + \frac{1}{2}$ $\qquad 1 + \frac{1}{1}$

$= [1; 1, 2]$

alternatively $= [1; 1, 1, 1]$ $\quad = a_n - 1 + \frac{1}{1}$ if $a_n > 1$

$1.5 = 1.499\ldots$ $\qquad [a_0; a_1, \ldots, a_n] = [a_0; a_1, \ldots, a_{n-1}, a_n - 1, 1]$

EG $\frac{43}{19} = 2 + \frac{5}{19}$ $\quad = [2; 3, 1, 4]$

repeat with $19/5$ ...

[or : $[2; 3, 1, 3, 1]$]

$43 = 2 \cdot 19 + 5$
$19 = 3 \cdot 5 + 4$
$5 = 1 \cdot 4 + 1$
$4 = 4 \cdot 1 + 0$