

Diophantine equations

only integer solutions

EG Find all solutions to the diophantine equation $16x + 25y = 3$.

①

$$\gcd(16, 25) = 1$$

Bezout's identity: $1 = 16r + 25s$

can find r, s using Euclid

$$3 = 16x + 25y$$

has solution $x = 3 \cdot 11 = 33$
 $y = 3 \cdot (-7) = -21$

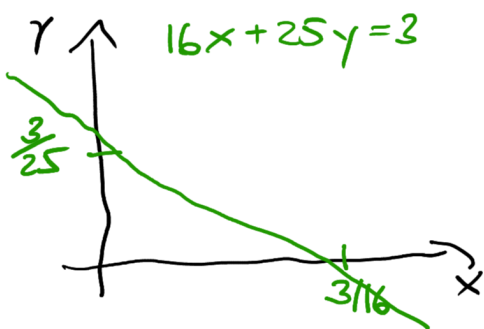
②

$$0 = 16 \cdot 25 + 25 \cdot (-16)$$

has further solutions:

$$\begin{aligned} x &= 33 + 25 \cdot t \\ y &= -21 - 16 \cdot t \end{aligned}$$

general solution



EG

$$6x + 15y = 10$$

$$\gcd(6, 15) = 3$$

$$\Leftrightarrow 2x + 5y = \frac{10}{3}$$

not an integer
must be integers

\leadsto no integer solutions

EG

$$32x + 50y = 6$$

$$\gcd(32, 50) = 2$$

$$\Leftrightarrow 16x + 25y = 3$$
$$\gcd(16, 25) = 1$$

solved above