

Extended Euclidean algorithm

EG Find $d = \gcd(16, 25)$ as well as integers r, s so that $d = 16r + 25s$.
Bezout's identity

$$25 = 2 \cdot 16 - 7$$

$$16 = 2 \cdot 7 + 2$$

$$7 = 3 \cdot 2 + 1$$

d

$$7 = 2 \cdot 16 - 25$$

$$\begin{aligned} 2 &= 1 \cdot 16 - 2 \cdot 7 \\ &= 1 \cdot 16 - 2 \cdot (2 \cdot 16 - 25) \\ &= -3 \cdot 16 + 2 \cdot 25 \end{aligned}$$

$$\begin{aligned} 1 &= 1 \cdot 7 - 3 \cdot 2 \\ &= (2 \cdot 16 - 25) - 3 \cdot (-3 \cdot 16 + 2 \cdot 25) \\ &= 11 \cdot 16 - 7 \cdot 25 \end{aligned}$$

r s