

Final: practice

MATH 311 — Intro to Number Theory
final exam: Tuesday, Dec 11

Please print your name:

As usual, calculators will not be permitted on the final exam. The numbers on the exam will be suitable for calculating by hand.

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches). Any typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Problem 1. The final exam will be comprehensive, that is, it will cover the material of the whole semester.

- (a) Do the practice problems for both midterms.
- (b) Retake both midterm exams. (The exams with and without solutions are posted.)
- (c) Do the problems below. (Solutions are posted.)

Problem 2.

- (a) State Wilson's theorem.
- (b) List all quadratic residues modulo 21.
- (c) What is the number of invertible quadratic residues modulo 101?
- (d) What is the number of invertible quadratic residues modulo 77?
- (e) Which real numbers have a finite continued fraction?

Problem 3.

- (a) Which number is represented by the continued fraction $[1; 2, 1, 2, 1, 2]$?
- (b) Determine all convergents of $[1; 2, 1, 2, 1, 2]$.
- (c) Which number is represented by the infinite continued fraction $[1; 2, 1, 2, 1, 2, 1, 2, \dots]$?
- (d) Compare, numerically, the first six convergents (computed above) to the value of the infinite continued fraction.

Problem 4.

- (a) Express the numbers $\frac{252}{193}$ and $-\frac{337}{221}$ as a simple continued fraction.
- (b) Is this the unique simple continued fraction representing $\frac{252}{193}$? Explain!