Midterm #1

MATH 316 — Linear Algebra II
Monday, Feb 23, 2026

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 32 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (7 points) Determine the QR decomposition of the matrix $\begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 2 & 1 \end{bmatrix}$.

Problem 2. (3 points) We want to find values for the parameters a, b, c such that $z = a + bx^2 + cy$ best fits some given points $(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots$ Set up a linear system such that $[a, b, c]^T$ is a least squares solution.

Armin Straub
straub@southalabama.edu

Problem 3. (6 points)

(a) Find the least squares solution to $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$.

(b) Determine the least squares line for the data points $(2,-1), (1,2), (1,0)$.

Problem 4. (4 points) Diagonalize the symmetric matrix $A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$ as $A = PDP^T$.

Problem 5. (4 points) Consider the vector space $W = \operatorname{span}\left\{\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}\right\}$.

(a) Determine the orthogonal projection of $\begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix}$ onto W.

(b) Determine the orthogonal projection of that same vector onto $W^\perp$.

Problem 6. (3 points) Let $A = \begin{bmatrix} 1 & 3 & 0 & 2 & 4 \\ 0 & 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) A basis for $\operatorname{null}(A)$ is $\square$. A basis for $\operatorname{col}(A)$ is $\square$.

(b) $\dim \operatorname{col}(A) = \square$, $\dim \operatorname{row}(A) = \square$, $\dim \operatorname{null}(A) = \square$, $\dim \operatorname{null}(A^T) = \square$

Problem 7. (5 points) Fill in the blanks.

(a) $\operatorname{col}(A)$ is the orthogonal complement of $\square$. $\operatorname{null}(A)$ is the orthogonal complement of $\square$.

(b) If A is a 4×8 matrix with rank 3, then $\dim \operatorname{null}(A) = \square$ and $\dim \operatorname{row}(A) = \square$.

(c) By definition, a matrix Q is orthogonal if and only if $\square$.

(d) The linear system $A\boldsymbol{x} = \boldsymbol{b}$ is consistent if and only if $\boldsymbol{b}$ is orthogonal to $\square$.

(e) Let W be the subspace of $\mathbb{R}^6$ of all solutions to $x_1 - x_4 + 3x_6 = 0$. $\dim W = \square$ and $\dim W^\perp = \square$.

Armin Straub
straub@southalabama.edu

(extra scratch paper)