

Homework Set 8

Problems 1 & 2

You find the answer to these problems at the beginning of Lecture 22.

Problem 3

Example 1. Let A be the matrix for reflecting through the plane spanned by $\begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$.

Diagonalize A as $A = PDP^{-1}$.

Solution. The eigenvalues of A are $1, 1, -1$. The 1 -eigenspace of A is W , and the -1 -eigenspace is W^\perp .

If $W = \text{span}\left\{\begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}\right\}$, then $W^\perp = \text{null}\left(\begin{bmatrix} 1 & 4 & -2 \\ 0 & 2 & -3 \end{bmatrix}\right)$.

It follows from $\begin{bmatrix} 1 & 4 & -2 \\ 0 & 2 & -3 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2 \Rightarrow R_2} \begin{bmatrix} 1 & 4 & -2 \\ 0 & 1 & -\frac{3}{2} \end{bmatrix} \xrightarrow{R_1 - 4R_2 \Rightarrow R_1} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -\frac{3}{2} \end{bmatrix}$ that $W^\perp = \text{span}\left\{\begin{bmatrix} -4 \\ 3/2 \\ 1 \end{bmatrix}\right\}$.

We can therefore choose $D = \begin{bmatrix} 1 & & \\ & 1 & \\ & & -1 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 0 & -4 \\ 4 & 2 & 3/2 \\ -2 & -3 & 1 \end{bmatrix}$.

Comment. From here, we can produce a diagonalization of the form $A = PDP^T$. How?