# Midterm #2

**MATH 316 — Linear Algebra II**
**Friday, Mar 29, 2024**

***Please print your name:***

No notes, calculators or tools of any kind are permitted. There are 30 points in total. You need to show work to receive full credit.

**Good luck!**

**Problem 1. (8 points)** Solve the initial value problem $\boldsymbol{y}' = \begin{bmatrix} 1 & -1 \\ -2 & 0 \end{bmatrix}\boldsymbol{y}$, $\boldsymbol{y}(0) = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$.

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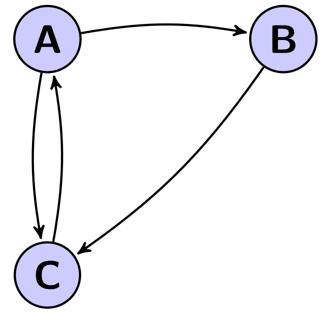
**Problem 2. (4 points)** Suppose the internet consists of only the three webpages $A, B$, $C$ which link to each other as indicated in the diagram.




Rank these webpages by computing their PageRank vector.

PageRank vector: [ ]. Ranking of websites: [ ].

**Problem 3. (8 points)** Fill in the blanks.

(a) Let $A$ be the $3 \times 3$ matrix for reflecting through a plane (containing the origin).

Then $\det(A) =$ [ ], and the eigenvalues (indicate if repeated) of $A$ are [ ].

(b) If $A$ is a reflection matrix, then $A^{2024} =$ [ ]. If $B$ is a projection matrix, then $B^{2024} =$ [ ].

(c) If $A$ has eigenvalue 3, then $A^2$ has eigenvalue [ ], $3A$ eigenvalue [ ], and $A^T$ eigenvalue [ ].

(d) If $A = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$, then $A^n =$ [ ] and $e^{At} =$ [ ].

(e) An example of a $2 \times 2$ matrix with eigenvalue $\lambda = 3$ that is not diagonalizable is [ ].

(f) If $N^3 = \mathbf{0}$, then $e^{Nt} =$ [ ].

(g) How many different Jordan normal forms are there in the following cases?

- A $5 \times 5$ matrix with eigenvalues $3, 3, 5, 5, 5$? [ ]
- A $9 \times 9$ matrix with eigenvalues $1, 2, 2, 2, 4, 4, 4, 4$? [ ]

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**Problem 4. (2 points)** Convert the third-order differential equation

$$y''' = 3y'' + 8y, \quad y(0) = 2, \quad y'(0) = 1, \quad y''(0) = -1$$

to a system of first-order differential equations.

**Problem 5. (1+4+1 points)** Consider the sequence $a_n$ defined by $a_{n+2} = 2a_{n+1} + 3a_n$ and $a_0 = 0$, $a_1 = 8$.

(a) The next two terms are $a_2 =$ ☐ and $a_3 =$ ☐.

(b) A Binet-like formula for $a_n$ is $a_n =$ ☐, and $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} =$ ☐.

**Problem 6. (2 points)** Let $A$ be the $3 \times 3$ matrix for reflecting through the plane spanned by the vectors $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

Determine an orthogonal matrix $P$ and a diagonal matrix $D$ such that $A = PDP^T$.

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(extra scratch paper)