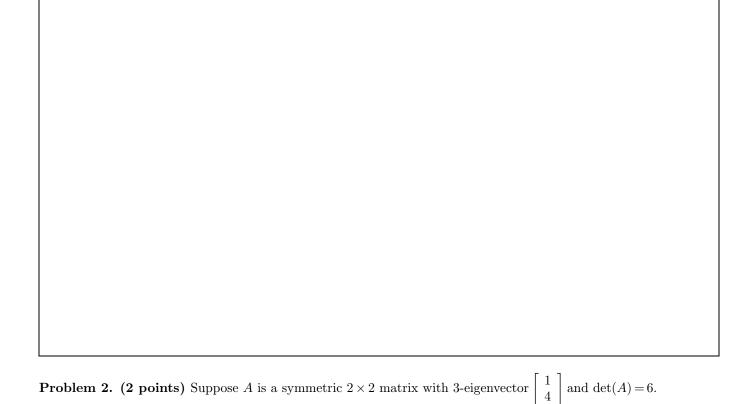
Please print your name:

No notes, calculators or tools of any kind are permitted. There are 31 points in total. You need to show work to receive full credit.

## Good luck!

Problem 1. (6 points)

- (a) Find the least squares solution to  $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} -2 \\ 0 \\ 5 \\ 2 \end{bmatrix}.$
- (b) Determine the least squares line for the data points (2, -2), (1, 0), (1, 5), (-1, 2).



Further,  $A = PDP^T$  with D =

-eigenvector

Then A has

and P =

## Problem 3. (9 points)

- (a) Using Gram–Schmidt, obtain an orthonormal basis for  $W = \operatorname{span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} \right\}$ .

  (b) Determine the orthogonal projection of  $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$  onto W.
- (c) Determine the orthogonal projection of that same vector onto  $W^{\perp}$ .
- (d) Determine the QR decomposition of the matrix  $\begin{bmatrix} 1 & 3 \\ 1 & 1 \\ 0 & -1 \end{bmatrix}$ .

**Problem 4.** (3 points) We want to find values for the parameters a, b, c such that  $z = ax + bx^2 + c\ln(y)$  best fits some given points  $(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots$  Set up a linear system such that  $[a, b, c]^T$  is a least squares solution.

Problem 5. (3 points) Let  $A = \begin{bmatrix} 1 & 5 & -2 & 0 & -4 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$ .

- (a) A basis for  $\operatorname{null}(A)$  is
- $(\mathrm{b}) \ \dim \mathrm{col}(A) = \boxed{\qquad}, \quad \dim \mathrm{null}(A) = \boxed{\qquad}, \quad \dim \mathrm{null}(A^T) = \boxed{\qquad}$

Problem 6. (8 points) Fill in the blanks.

- (a)  $\operatorname{null}(A)$  is the orthogonal complement of  $\operatorname{col}(A)$  is the orthogonal complement of
- (b) If A is a  $5 \times 7$  matrix with rank 4, then  $\dim \operatorname{col}(A) = \square$  and  $\dim \operatorname{null}(A) = \square$
- (c) By definition, a matrix Q is orthogonal if and only if
- (d) If Q is orthogonal, then det(Q) is
- (e) The linear system Ax = b is consistent if and only if b is orthogonal to
- (f) For which matrices A is it true that  $A^{-1} = A^{T}$ ?
- (g) The projection matrix for orthogonally projecting onto  $\operatorname{col}(A)$  is P = . If A has orthonormal columns, this simplifies to

(extra scratch paper)