Midterm #1

MATH 316 — Linear Algebra II
Friday, Feb 16, 2024

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 31 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (6 points)

(a) Find the least squares solution to $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} -2 \\ 0 \\ 5 \\ 2 \end{bmatrix}$.

(b) Determine the least squares line for the data points $(2, -2), (1, 0), (1, 5), (-1, 2)$.

Problem 2. (2 points) Suppose A is a symmetric 2×2 matrix with 3-eigenvector $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $\det(A) = 6$.

Then A has ☐-eigenvector ☐. Further, $A = PDP^T$ with $D =$ ☐ and $P =$ ☐.

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Problem 3. (9 points)

(a) Using Gram–Schmidt, obtain an orthonormal basis for $W = \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} \right\}$.

(b) Determine the orthogonal projection of $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ onto W.

(c) Determine the orthogonal projection of that same vector onto $W^{\perp}$.

(d) Determine the QR decomposition of the matrix $\begin{bmatrix} 1 & 3 \\ 1 & 1 \\ 0 & -1 \end{bmatrix}$.

Problem 4. (3 points) We want to find values for the parameters a, b, c such that $z = ax + bx^2 + c\ln(y)$ best fits some given points $(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots$ Set up a linear system such that $[a, b, c]^T$ is a least squares solution.

Problem 5. (3 points) Let $A = \begin{bmatrix} 1 & 5 & -2 & 0 & -4 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$.

(a) A basis for null(A) is ______. A basis for col(A) is ______.

(b) $\dim \operatorname{col}(A) =$ ______, $\dim \operatorname{row}(A) =$ ______, $\dim \operatorname{null}(A) =$ ______, $\dim \operatorname{null}(A^T) =$ ______

Problem 6. (8 points) Fill in the blanks.

(a) null(A) is the orthogonal complement of ______. col(A) is the orthogonal complement of ______.

(b) If A is a 5×7 matrix with rank 4, then $\dim \operatorname{col}(A) =$ ______ and $\dim \operatorname{null}(A) =$ ______.

(c) By definition, a matrix Q is orthogonal if and only if ______.

(d) If Q is orthogonal, then $\det(Q)$ is ______.

(e) The linear system $A\boldsymbol{x} = \boldsymbol{b}$ is consistent if and only if $\boldsymbol{b}$ is orthogonal to ______.

(f) For which matrices A is it true that $A^{-1} = A^T$? ______

(g) The projection matrix for orthogonally projecting onto col(A) is $P =$ ______.

If A has orthonormal columns, this simplifies to ______.

(h) Let W be the subspace of $\mathbb{R}^5$ of all solutions to $x_1 - x_3 + 2x_5 = 0$. $\dim W =$ ______ and $\dim W^{\perp} =$ ______.

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(extra scratch paper)