Midterm #2

MATH 316 — Linear Algebra II
Friday, Mar 31, 2023

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 30 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (8 points) Solve the initial value problem $\boldsymbol{y}' = \begin{bmatrix} 1 & 3 \\ -1 & 5 \end{bmatrix} \boldsymbol{y}, \quad \boldsymbol{y}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

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Problem 2. (1+4+1 points) Consider the sequence a_n defined by $a_{n+2} = a_{n+1} + 2a_n$ and $a_0 = 1$, $a_1 = 8$.

(a) The next two terms are $a_2 = \square$ and $a_3 = \square$.

(b) A Binet-like formula for a_n is $a_n = \square$, and $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \square$.

Problem 3. (2 points) Let A be the 3×3 matrix for reflecting through the plane spanned by the vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$. Determine an orthogonal matrix P and a diagonal matrix D such that $A = PDP^T$.

Problem 4. (1+1+2 points) Fill in the blanks.

(a) An example of a 2×2 matrix with eigenvalue $\lambda = 5$ that is not diagonalizable is $\square$.

(b) If $N^3 = \mathbf{0}$, then $e^{Nt} = \square$.

(c) How many different Jordan normal forms are there in the following cases?

- A 4×4 matrix with eigenvalues $2, 5, 5, 5$? $\square$
- A 8×8 matrix with eigenvalues $1, 1, 2, 2, 4, 4, 4, 4$? $\square$

Problem 5. (4 points) Fill in the blanks.

(a) Let A be the 4×4 matrix for orthogonally projecting onto a 2-dimensional subspace of $\mathbb{R}^4$.

Then $\det(A) =$ ______, and the eigenvalues (indicate if repeated) of A are ______.

(b) If A is a projection matrix, then $A^{2024} =$ ______. If B is a reflection matrix, then $B^{2024} =$ ______.

(c) If A has eigenvalue 2, then A^3 has eigenvalue ______, $3A$ eigenvalue ______, and A^T eigenvalue ______.

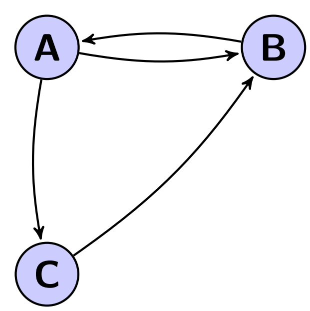
(d) If $A = \begin{bmatrix} -2 & 0 \\ 0 & 4 \end{bmatrix}$, then $A^n =$ ______ and $e^{At} =$ ______.

Problem 6. (2 points) Convert the third-order differential equation

$$y''' = 6y'' - 3y' - 10y, \quad y(0) = 1, \quad y'(0) = 2, \quad y''(0) = 3$$

to a system of first-order differential equations.

Problem 7. (4 points) Suppose the internet consists of only the three webpages A, B, C which link to each other as indicated in the diagram.



Rank these webpages by computing their PageRank vector.

PageRank vector: ______. Ranking of websites: ______.

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(extra scratch paper)

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