Preparing for Midterm #2

MATH 316 — Linear Algebra II
Midterm: Friday, Mar 31, 2023

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches or any other posted material). Any typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Problem 1. Before working on the problems below, make sure that you have completed all homework. (In particular, Homework Sets 6, 7, 8 and 9.)

Problem 2. Solve the initial value problem $\boldsymbol{y}' = \begin{bmatrix} 4 & -8 \\ -1 & 6 \end{bmatrix} \boldsymbol{y}, \quad \boldsymbol{y}(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

(Use Homework Problem 8.5 to generate more practice problems of this kind.)

Problem 3.

(a) Convert the third-order differential equation

$$y''' = 6y'' - 3y' - 10y, \quad y(0) = 1, \quad y'(0) = 2, \quad y''(0) = 3$$

to a system of first-order differential equations.

(b) Solve the original differential equation by solving the system.

(Use Homework Problems 8.6, 8.8 to generate more practice problems of this kind.)

Problem 4.

(a) What are the possible Jordan normal forms of a 6×6 matrix with eigenvalues $7, 7, 3, 3, 3, 3$?

(b) How many different Jordan normal forms are there for a 10×10 matrix with eigenvalues $8, 6, 6, 2, 2, 2, 1, 1, 1, 1$?

(Use Homework Problems 9.1, 9.2 to generate more practice problems of this kind.)

Problem 5. Consider the sequence a_n defined by $a_{n+2} = 4a_{n+1} - a_n$ and $a_0 = 1$, $a_1 = 0$.

(a) Determine the next three terms.

(b) A Binet-like formula for a_n is $a_n =$ [ ].

(c) $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} =$ [ ].

(Use Homework Problems 7.1, 7.2, 7.3, 7.4, 7.5 to generate more practice problems of this kind.)

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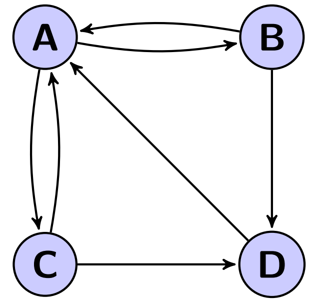
Problem 6. Suppose the internet consists of only the four webpages A, B, C, D which link to each other as indicated in the diagram.



Rank these webpages by computing their PageRank vector.

(Use Homework Problem 6.6 to generate more practice problems of this kind.)

Problem 7. Determine an orthogonal matrix P and a diagonal matrix D such that $A = PDP^T$.

(a) If A is the 3×3 matrix for reflecting through the plane spanned by the vectors $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

(b) If A is the 3×3 matrix for reflecting through the plane spanned by the vectors $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

(Use Homework Problem 8.2 to generate more practice problems of this kind.)

Problem 8.

(a) The eigenvalues of a 5×5 matrix for orthogonally projecting onto a 3-dimensional subspace are $\boxed{}$.

What are the eigenspaces of that matrix?

(b) Suppose A is the 3×3 matrix of a reflection through a plane (containing the origin).

Then $\det(A) = \boxed{}$, and the eigenvalues of A are $\boxed{}$. What are the eigenspaces of A?

(c) Precisely state the spectral theorem.

(d) If A is a reflection matrix, then $A^{-1} = \boxed{}$.

(e) If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then $A^n = \boxed{}$ and $e^{At} = \boxed{}$.

(f) If $N^4 = \mathbf{0}$, then $e^{Nt} = \boxed{}$.

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