Final Exam

MON, 5/3

exam: 8:00 – 10:00 AM upload work by 10:30 AM

PDF

format

- like for midterm exams
- show-your-work problems ~ 4
- short answer problems ~ 16
  - no "real" work needed

practice

- review HW
- practice problems + solutions

tools

- calculators allowed but: show work
- notes allowed but: watch time

Questions?

- Lots of review
  - Gaussian elimination, null spaces, determinants, eigenstuff, ...

not on exam

- LU decomposition
- diagonalization

THEME: **orthogonality**

- fundamental theorem
  - col(A) and null(A^T) are orthogonal complements
- consistency of linear systems
- least squares solutions
  - application: least squares lines (linear regression)
  - application: data fitting
- orthogonal projections
  - projection matrices
  - using orthogonal bases
- Gram-Schmidt + QR
- orthogonal matrices
- diagonalizability
- spectral theorem

THEME: diagonalizations

$A = PDP^{-1}$

$A^n = PD^nP^{-1}$

- powers of matrix
- application: Markov chains
- application: PageRank
- application: recurrences
  - Binet-like formulas
  - asymptotic growth ($\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$)
- application: systems of DEs

  $e^{At} = Pe^{Dt}P^{-1}$
  - matrix exponential
  - convert higher-order DEs to systems
- non-diagonalizable $\rightsquigarrow$ Jordan normal form
  - EG $e^{\begin{pmatrix}7 & 1\\ 0 & 7\end{pmatrix}t}$
  - # of possible JNFs
- projections, reflections, rotations
- complex numbers
  - transpose A^T $\rightsquigarrow$ conjugate transpose A^*

THEME: SVD + best approximations

$A = U\Sigma V^T$

- SVD
- pseudoinverses
- matrix approximations
  - lossy compression
- function spaces
  - natural dot product
- function approximation
  - via orthogonal projections + Gram-Schmidt
  - application: Fourier series