Fourier series

$$f(x) = a_0 + a_1 \cos(x) + b_1 \sin(x)$$
$$+ a_2 \cos(2x) + b_2 \sin(2x) + \ldots$$

f 2π-periodic $\quad f : [0, 2\pi] \to \mathbb{R}$

natural dot product:

$$\langle f, g \rangle = \int_0^{2\pi} f(t) g(t)\, dt$$

THM $1, \cos(x), \sin(x), \cos(2x), \sin(2x), \ldots$ are all orthogonal to each other.

actually: orthogonal basis

EG $\langle \cos(x), \sin(2x) \rangle = \int_0^{2\pi} \cos(t) \sin(2t)\, dt = 0$

Calc II

EG norm of $\cos(x)$? $\sqrt{\pi}$

$$\langle \cos(x), \cos(x) \rangle = \int_0^{2\pi} \cos^2(t)\, dt = \pi$$

How to compute Fourier series?

$a_n \cos(nx)$ = orth. proj. of f onto $\cos(nx)$

$$= \frac{\langle f(x), \cos(nx) \rangle}{\langle \cos(nx), \cos(nx) \rangle} \cos(nx)$$

π

$$\Rightarrow a_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(nt)\, dt$$