Function spaces

functions $f:[a,b] \to \mathbb{R}$ (piecewise) continuous

vector space dim $= \infty$

vectors?
- add
- scale

EG functions!

natural dot product

$$\langle f, g \rangle := \int_a^b f(t)\, g(t)\, dt$$

$$\langle \vec{x}, \vec{y} \rangle = x_1 y_1 + x_2 y_2 + \dots = \sum_{t=1}^{n} x_t y_t$$

EG What is the orthogonal projection of $f:[a,b] \to \mathbb{R}$ onto span$\{1\}$?

projection of x onto y

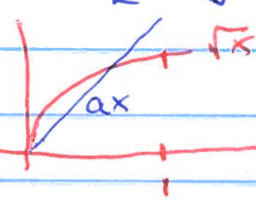
$$\frac{\langle x, y \rangle}{\langle y, y \rangle} y$$

$$= \frac{\langle f, 1 \rangle}{\langle 1, 1 \rangle} 1 = \frac{\int_a^b f(t) \cdot 1\, dt}{\int_a^b 1 \cdot 1\, dt} \cdot 1$$

$$= \frac{1}{b-a} \int_a^b f(t)\, dt \quad = \text{average of } f \text{ on } [a,b]$$

"best constant approx. of f"

EG Best approx. of $f(x) = \sqrt{x}$ on $[0,1]$ using a function $y = ax$.



$=$ orth. proj. of $\sqrt{x}$ onto span$\{x\}$

$$= \frac{\langle \sqrt{x}, x \rangle}{\langle x, x \rangle} x = \frac{\int_0^1 \sqrt{t} \cdot t\, dt}{\int_0^1 t \cdot t\, dt} x = \frac{2/5}{1/3} x = \underline{\underline{\frac{6}{5} x}}$$

$$\int t^a\, dt = \frac{1}{a+1} t^{a+1}$$

$$\int_0^1 t^2\, dt = \left[\frac{1}{3} t^3\right]_0^1 = \frac{1}{3} 1^3 - \frac{1}{3} 0^3 = \frac{1}{3}$$

$$\int_0^1 t^{3/2}\, dt = \left[\frac{2}{5} t^{5/2}\right]_0^1 = \frac{2}{5} \cdot 1^{5/2} - \frac{2}{5} 0^{5/2} = \frac{2}{5}$$