Complex numbers

$z = x + iy$

i solves $x^2 = -1$

so does $-i$

$\bar{z} = x - iy$ <u>conjugate</u>

$$|z| = \sqrt{x^2 + y^2} = \sqrt{\bar{z} z}$$

absolute value

$$\left\| \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \right\| = \sqrt{|z_1|^2 + |z_2|^2}$$

norm

EG $$\left\| \begin{bmatrix} 1-i \\ 2+3i \end{bmatrix} \right\| = \sqrt{|1-i|^2 + |2+3i|^2}$$
$$= \sqrt{(1^2 + (-1)^2) + (2^2 + 3^2)} = \sqrt{1+1+4+9}$$
$$= \sqrt{15}$$

$$\left\| \begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \end{bmatrix} \right\|$$

A^H, A^+

DEF $A^* = \bar{A}^T$ conjugate transpose

EG $A = \begin{bmatrix} 2 & 1-i \\ 3+2i & i \end{bmatrix}$ $A^* = \begin{bmatrix} 2 & 1+i \\ 3-2i & -i \end{bmatrix}^T = \begin{bmatrix} 2 & 3-2i \\ 1+i & -i \end{bmatrix}$

EG $z^* z = \begin{bmatrix} \bar{z}_1 & \bar{z}_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \underbrace{\bar{z}_1 z_1}_{|z_1|^2} + \underbrace{\bar{z}_2 z_2}_{|z_2|^2} = \|z\|^2$

$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

<u>dot product</u> $v \cdot w \underset{\text{DEF}}{=} v^* w$

<u>Euler's identity</u>

$$e^{it} = \cos(t) + i \sin(t)$$

$t = \pi$: $e^{i\pi} = -1$

Pf both sides solve $y' = iy$ $y(0) = 1$

$y(t) = \cos(t) + i \sin(t)$

$y'(t) = -\sin(t) + i \cos(t) = i [\cos(t) + i \sin(t)] = i\, y(t)$

$y(0) = \cos(0) + i \underbrace{\sin(0)}_{0} = 1$