$J = \begin{bmatrix} J_1 & & \\ & J_2 & \\ & & \ddots \end{bmatrix}$

$J_i = \begin{bmatrix} \lambda_i & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & \lambda_i \end{bmatrix}$

unique up to ordering of J_i

Eg What are the possible Jordan normal forms?

(a) for a 2×2 matrix with eigenvalues 7,7

$\begin{bmatrix} 7 & \\ & 7 \end{bmatrix}$ $\begin{bmatrix} 7 & 1 \\ & 7 \end{bmatrix}$ #=2

(b) for a 3×3 matrix with eigenvalues 7,7,7

$\begin{bmatrix} 7 & & \\ & 7 & \\ & & 7 \end{bmatrix}$ $\begin{bmatrix} 7 & & \\ & 7 & 1 \\ & & 7 \end{bmatrix}$ $\begin{bmatrix} 7 & 1 & \\ & 7 & 1 \\ & & 7 \end{bmatrix}$ #=3

1+1+1 1+2 3

equivalent $\begin{bmatrix} 7 & 1 & \\ & 7 & \\ & & 7 \end{bmatrix}$

(c) for a 4×4 matrix with eigenvalues 7,7,7,7

1+1+1+1 1+1+2 1+3 2+2 4 #=5

(d) for a 5×5 matrix with eigenvalues 7,7,7,7,7

1+1+1+1+1 1+1+1+2 1+1+3

1+2+2 1+4 2+3 5 #7

$\begin{bmatrix} 7 & & & & \\ & 7 & 1 & & \\ & & 7 & & \\ & & & 7 & 1 \\ & & & & 7 \end{bmatrix}$