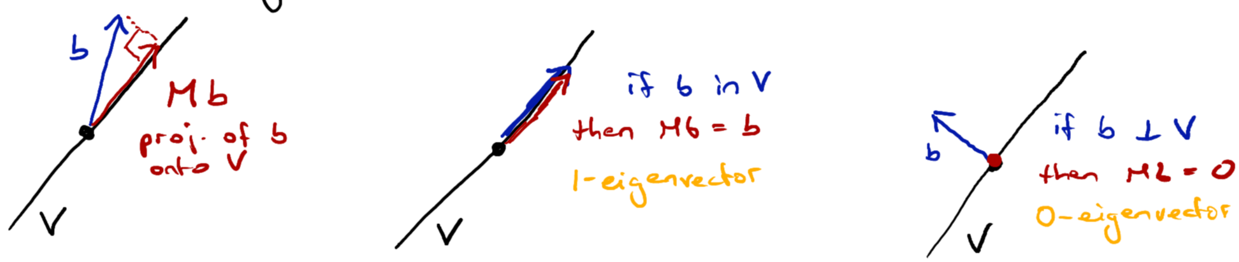
Projection matrices — a 2nd look

M projection matrix for (orth.) projecting onto V

earlier $= A(A^TA)^{-1}A^T$ where $V = \mathrm{col}(A)$

THM (eigenspaces of proj. matrices) The 1-eigenspace of M is V.
The 0-eigenspace of M is $V^\perp$.



in particular eigenspaces are orthogonal

$\xrightarrow[\text{theorem}]{\text{spectral}}$ projection matrices are symmetric

EG M = matrix for (orthogonally) projecting onto
$$V = \mathrm{span}\left\{\begin{bmatrix}4\\0\\1\end{bmatrix}, \begin{bmatrix}0\\2\\1\end{bmatrix}\right\}$$

earlier computed M as $M = A(A^TA)^{-1}A^T$ with $A = \begin{bmatrix}4 & 0\\0 & 2\\1 & 1\end{bmatrix}$

now make statements about M without computing it

(a) Is M invertible? NO Orthogonal? NO Symmetric? YES

(b) Diagonalize M.

$$M = PDP^{-1} \quad \text{with} \quad D = \begin{bmatrix}1 & & \\ & 1 & \\ & & 0\end{bmatrix}$$

0-eigenspace of M = $V^\perp$

$= \mathrm{null}\left(\begin{bmatrix}4 & 0 & 1\\0 & 2 & 1\end{bmatrix}\right)$

$$P = \begin{bmatrix}4 & 0 & \boxed{-1/4}\\0 & 2 & \boxed{-1/2}\\1 & 1 & \boxed{1}\end{bmatrix}$$

basis: $\begin{bmatrix}-1/4\\-1/2\\1\end{bmatrix}$

HW Diagonalize M as $M = PDP^T$