

Orthogonal bases: examples

review orthogonal proj. \hat{w} of w onto V
 if v_1, v_2, \dots, v_n is an orthogonal basis of V

$$= \frac{w \cdot v_1}{v_1 \cdot v_1} v_1 + \dots + \frac{w \cdot v_n}{v_n \cdot v_n} v_n$$

orth. proj. of w onto v_1
orth. proj. of w onto v_n

EG Express $\begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix}$ in the basis $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

$$= c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \text{not orthogonal}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix} \quad \begin{array}{l} \text{Gaussian} \\ \text{elimination} \end{array} \quad \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$$

EG Express $\begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix}$ in the basis $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$= \frac{w \cdot v_1}{v_1 \cdot v_1} v_1 + \frac{w \cdot v_2}{v_2 \cdot v_2} v_2 + \frac{w \cdot v_3}{v_3 \cdot v_3} v_3 \quad \text{orthogonal!}$$

$$= \frac{-4}{2} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \frac{10}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \frac{4}{1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$c_1 = -2$
 $c_2 = 5$
 $c_3 = 4$

EG Orthogonally project $\begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix}_w$ onto $\text{span} \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

v_1 v_2
orthogonal!

$$\text{projection} = \frac{w \cdot v_1}{v_1 \cdot v_1} v_1 + \frac{w \cdot v_2}{v_2 \cdot v_2} v_2$$

$$= \frac{-4}{2} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \frac{10}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 0 \end{bmatrix}$$

check $\begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ 7 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}$ needs to be orthogonal to v_1, v_2
 "error"