Projection matrices

recall $\hat{b}$ = orth. proj. of b onto $\text{col}(A)$

$= A\hat{x}$ where $\hat{x}$ is least squares solution to $Ax = b$

$\rightsquigarrow A^TA\hat{x} = A^Tb$

$\rightsquigarrow \hat{x} = (A^TA)^{-1}A^Tb$

A^TA invertible $\Leftrightarrow$ cols of A independent

$$\hat{b} = A(A^TA)^{-1}A^Tb$$

projection matrix for projecting onto $\text{col}(A)$

EG Matrix for projecting onto $\text{span}\left\{\begin{bmatrix}4\\0\\1\end{bmatrix}, \begin{bmatrix}0\\2\\1\end{bmatrix}\right\} = \text{col}(A)$

$$A = \begin{bmatrix}4 & 0\\ 0 & 2\\ 1 & 1\end{bmatrix}$$

$$P = A(A^TA)^{-1}A^T$$

$$= \begin{bmatrix}17 & 1\\ 1 & 5\end{bmatrix}^{-1} = \frac{1}{84}\begin{bmatrix}5 & -1\\ -1 & 17\end{bmatrix}$$

$$= \frac{1}{21}\begin{bmatrix}20 & -2 & 4\\ -2 & 17 & 8\\ 4 & 8 & 5\end{bmatrix}$$

observations

- orth. proj. of $\begin{bmatrix}2\\0\\11\end{bmatrix}$ is $P\begin{bmatrix}2\\0\\11\end{bmatrix} = \begin{bmatrix}4\\4\\3\end{bmatrix}$
- the columns of P are the projections of the standard basis vectors

  first col $= \frac{1}{21}\begin{bmatrix}20\\-2\\4\end{bmatrix}$ = proj. of $\begin{bmatrix}1\\0\\0\end{bmatrix}$
- $P^2 = P$
- the orth. proj. of v onto $\omega = \frac{\omega \cdot v}{\|\omega\|^2}\omega$

  $\hat{v} = \omega\underbrace{(\omega^T\omega)^{-1}}_{\frac{1}{\|\omega\|^2}}\underbrace{\omega^Tv}_{\omega\cdot v}$