Orthogonality

inner product (dot product)

$$v \cdot w = v_1 w_1 + v_2 w_2 + \cdots$$
$$= v^T w$$

EG $\begin{bmatrix}1\\2\\3\end{bmatrix} \cdot \begin{bmatrix}2\\-1\\4\end{bmatrix} = 1\cdot 2 + 2\cdot(-1) + 3\cdot 4 = 12$

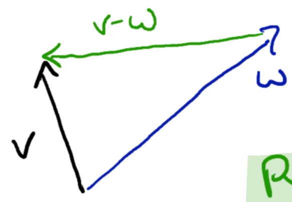
norm (length)

$$\|v\| = \sqrt{v_1^2 + v_2^2 + \cdots}$$
$$= \sqrt{v \cdot v}$$

EG simplify $\|v-w\|^2 = (v-w)\cdot(v-w) = v\cdot v - v\cdot w - \overbrace{w\cdot v}^{v\cdot w} + w\cdot w$

$$= \|v\|^2 - 2\, v\cdot w + \|w\|^2$$

vector version of $(a-b)^2 = a^2 - 2ab + b^2$



$\|v\|$, $\|w\|$, $\|v-w\|$ sides of a triangle!

Pythagoras v, w orthogonal

$$\iff \underbrace{\|v\|^2}_{a^2} + \underbrace{\|w\|^2}_{b^2} = \underbrace{\|v-w\|^2}_{c^2}$$
$$= \|v\|^2 - 2 v\cdot w + \|w\|^2$$

$$\iff 0 = -\cancel{2}\, v\cdot w$$

orthogonal v, w orthogonal $\iff v\cdot w = 0$

EG find vector orthogonal to $\begin{bmatrix}2\\3\end{bmatrix}$

$\begin{bmatrix}3\\-2\end{bmatrix}$ (or any multiple)

$\begin{bmatrix}2\\3\end{bmatrix}\cdot\begin{bmatrix}3\\-2\end{bmatrix} = 2\cdot 3 + 3\cdot(-2) = 0$

in general $\begin{bmatrix}a\\b\end{bmatrix}$ orthogonal to $\begin{bmatrix}b\\-a\end{bmatrix}$