Crash course: Determinants

determinant of A: $\det(A)$, $|A|$

$\det(A) \neq 0 \iff A$ is invertible
$\iff Ax = b$ has a unique sol. (for every b)

$x = A^{-1}b$

EG $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

EG Compute $\begin{vmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \\ 2 & 0 & 1 \end{vmatrix}$ by cofactor expansion.

Solution expand by 1st row

$$\begin{vmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = +1 \cdot \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} - 2 \cdot \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} + 0 \cdot \ldots$$

$$\begin{matrix} + & - & \cdots \\ - & + & \cdots \\ \vdots & & \end{matrix}$$

$$= 1 \cdot (-1) - 2(3 \cdot 1 - 2 \cdot 2) = -1 + 2 = 1$$

Solution expand by 2nd column

$$\begin{vmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = -2 \cdot \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} + (-1) \cdot \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} - 0 \cdot \ldots$$

$$= -2(-1) + (-1) \cdot 1 = 2 - 1 = 1$$

EG

$$\begin{vmatrix} 1 & 0 & 3 & 4 \\ 0 & 2 & 1 & 5 \\ 0 & 0 & 2 & 1 \\ 2 & 0 & 8 & 5 \end{vmatrix} = +2 \cdot \begin{vmatrix} 1 & 3 & 4 \\ 0 & 2 & 1 \\ 2 & 8 & 5 \end{vmatrix}$$

$$= 2 \left[-0 \cdot \begin{vmatrix} 3 & 4 \\ 8 & 5 \end{vmatrix} + 2 \cdot \begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & 3 \\ 2 & 8 \end{vmatrix} \right]$$

$$= 2 \left[0 + 2 \cdot (-3) - 1 \cdot 2 \right] = 2 \cdot (-6 - 2) = -16$$