LU decomposition

$$\underset{m\times n}{A} = \underset{m\times m}{L}\ \underset{m\times n}{U}$$

U upper triangular

L lower triangular, invertible (1's on diagonal)

EG

$$A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 3 & 2 & 7 & 2 \\ -2 & 6 & -3 & 1 \end{bmatrix} \xrightarrow[\substack{R_2 - 3R_1 \Rightarrow R_2 \\ R_3 + 2R_1 \Rightarrow R_3}]{E} \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & 8 & 1 & 3 \end{bmatrix} = EA$$

$$\xrightarrow[R_3 + 8R_2 \Rightarrow R_3]{F} \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 9 & -5 \end{bmatrix} = FEA$$

U echelon form (upper triangular)

$$L = \begin{bmatrix} 1 & & \\ 3 & 1 & \\ -2 & -8 & 1 \end{bmatrix}$$

Why?

$$E = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 8 & 1 \end{bmatrix}$$

$$F^{-1}FEA = F^{-1}U$$

$$E^{-1}EA = E^{-1}F^{-1}U$$

$(FE)^{-1} = E^{-1}F^{-1}$

$$A = E^{-1}F^{-1}U = \begin{bmatrix} 1 & & \\ 3 & 1 & \\ -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & & \\ 0 & 1 & \\ 0 & -8 & 1 \end{bmatrix} U$$

$$= \begin{bmatrix} 1 & & \\ 3 & 1 & \\ -2 & -8 & 1 \end{bmatrix} U = LU$$

EG

$$A = \begin{bmatrix} 2 & 1 \\ 4 & -6 \end{bmatrix} \xrightarrow[R_2 - 2R_1 \Rightarrow R_2]{E} \begin{bmatrix} 2 & 1 \\ 0 & -8 \end{bmatrix} = U$$

EA

$$A = LU$$

$$L = \begin{bmatrix} 1 & \\ 2 & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$EA = U$$

$$A = E^{-1}U$$

L

$R_2 + 2R_1 \Rightarrow R_2$