Review: Gaussian elimination

EG

$$\begin{bmatrix} \textcircled{1} & 1 & 2 & 1 \\ 3 & 2 & 7 & 2 \\ -2 & 6 & -3 & 1 \end{bmatrix} \xrightarrow{\substack{R_2 - 3R_1 \Rightarrow R_2 \\ R_3 + 2R_1 \Rightarrow R_3}} \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & \textcircled{-1} & 1 & -1 \\ 0 & 8 & 1 & 3 \end{bmatrix}$$

pivot: first non-zero entries in each row

$$\xrightarrow{R_3 + 8R_2 \Rightarrow R_3} \begin{bmatrix} \textcircled{1} & 1 & 2 & 1 \\ 0 & \textcircled{-1} & 1 & -1 \\ 0 & 0 & \textcircled{9} & -5 \end{bmatrix}$$

echelon form

- zeros below pivots

RREF row-reduced echelon form

- zeros below + above pivots
- all pivots are 1

$$\begin{bmatrix} 1 & * & 0 & * & * & 0 & * \\ 0 & 0 & 1 & * & * & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 1 & * \end{bmatrix}$$

$$\xrightarrow{\substack{-R_2 \Rightarrow R_2 \\ \frac{1}{9}R_3 \Rightarrow R_3}} \begin{bmatrix} \textcircled{1} & 1 & 2 & 1 \\ 0 & \textcircled{1} & -1 & 1 \\ 0 & 0 & \textcircled{1} & -\frac{5}{9} \end{bmatrix}$$

$$\xrightarrow{\substack{R_1 - 2R_3 \Rightarrow R_1 \\ R_2 + R_3 \Rightarrow R_2}} \begin{bmatrix} \textcircled{1} & 1 & 0 & \frac{19}{9} \\ 0 & \textcircled{1} & 0 & \frac{4}{9} \\ 0 & 0 & \textcircled{1} & -\frac{5}{9} \end{bmatrix}$$

RREF is unique !

$$\xrightarrow{R_1 - R_2 \Rightarrow R_1} \begin{bmatrix} 1 & 0 & 0 & \frac{5}{3} \\ 0 & 1 & 0 & 4/9 \\ 0 & 0 & 1 & -5/9 \end{bmatrix}$$

Q Why row operations? Why RREF?

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 3 & 2 & 7 & 2 \\ -2 & 6 & -3 & 1 \end{array}\right]$$

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 1 \\ 3x_1 + 2x_2 + 7x_3 &= 2 \\ -2x_1 + 6x_2 - 3x_3 &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5/3 \\ 0 & 1 & 0 & 4/9 \\ 0 & 0 & 1 & -5/9 \end{array}\right]$$

$$\begin{aligned} x_1 &= 5/3 \\ x_2 &= 4/9 \\ x_3 &= -5/9 \end{aligned}$$