Crash course in linear algebra, II

EG $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 2×2 identity

$2\cdot 1 + 1\cdot(-2)$

$AB = I \rightsquigarrow B = A^{-1}$

A^{-1} = (matrix) inverse of A

characterized by $A^{-1}A = I = AA^{-1}$

$$\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$ad-bc$: determinant

provided if $ad - bc \neq 0$

EG

$$\begin{aligned} 7x_1 - 2x_2 &= 3 \\ 2x_1 + x_2 &= 5 \end{aligned} \quad \Longleftrightarrow \quad \begin{bmatrix} 7 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$A^{-1}A\vec{x} = A^{-1}\vec{b}$

$x = A^{-1}b$

$$\vec{x} = \begin{bmatrix} 7 & -2 \\ 2 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 1 & 2 \\ -2 & 7 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 13 \\ 29 \end{bmatrix}$$

$7\cdot 1 - (-2)\cdot 2$

$x_1 = \frac{13}{11}$

$x_2 = \frac{29}{11}$

$\det(A)$ = determinant of A

or: $|A|$

$\det(A) \neq 0$ (number) $\Longleftrightarrow$ A is invertible

$\Longleftrightarrow$ $Ax = b$ has a (unique) solution for all b

$x = A^{-1}b$

EG $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$