# Quiz #1

**MATH 316 — Linear Algebra II**
**Monday, Feb 17**

***Please print your name:***

**Problem 1.** We want to find values for the parameters $a, b, c$ such that $z = a + bx + c\ln(y)$ best fits some given points $(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots$ Set up a linear system such that $[a, b, c]^T$ is a least squares solution.

**Solution.** The equations $a + bx_i + b\ln(y_i) = z_i$ translate into the system:

$$\underbrace{\begin{bmatrix} 1 & x_1 & \ln(y_1) \\ 1 & x_2 & \ln(y_2) \\ 1 & x_3 & \ln(y_3) \\ \vdots & \vdots & \vdots \end{bmatrix}}_{A} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \end{bmatrix}}_{\boldsymbol{z}}$$

Of course, this is usually inconsistent. To find the best possible $a, b, c$ we compute a least squares solution.

**Problem 2.** Let $A = \begin{bmatrix} 1 & 5 & -2 & 0 & -4 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$.

(a) A basis for null($A$) is $\boxed{\phantom{xxxxxxxx}}$. A basis for col($A$) is $\boxed{\phantom{xxxxxxxx}}$.

(b) $\dim \operatorname{col}(A) = \boxed{\phantom{xx}}$, $\dim \operatorname{row}(A) = \boxed{\phantom{xx}}$, $\dim \operatorname{null}(A) = \boxed{\phantom{xx}}$, $\dim \operatorname{null}(A^T) = \boxed{\phantom{xx}}$

**Solution.**

(a) A basis for null($A$) is $\begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 0 \\ -3 \\ 1 \end{bmatrix}$. A basis for col($A$) is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(b) $\dim \operatorname{col}(A) = 2$, $\dim \operatorname{row}(A) = 2$, $\dim \operatorname{null}(A) = 3$, $\dim \operatorname{null}(A^T) = 0$

**Problem 3.** Fill in the blanks.

(a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \boxed{\phantom{xxxxxxxx}}$

(b) $\hat{\boldsymbol{x}}$ is a least squares solution of $A\boldsymbol{x} = \boldsymbol{b}$ if and only if $\boxed{\phantom{xxxxxxxx}}$.

(c) col($A$) is the orthogonal complement of $\boxed{\phantom{xxxx}}$. null($A$) is the orthogonal complement of $\boxed{\phantom{xxxx}}$.

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(d) The linear system $A\boldsymbol{x}=\boldsymbol{b}$ is consistent if and only if $\boldsymbol{b}$ is orthogonal to $\boxed{\phantom{xxxxxxxx}}$.

(e) The projection matrix for orthogonally projecting onto $\operatorname{col}(A)$ is $P=\boxed{\phantom{xxxxxxxx}}$.

[We assume that the columns of $A$ are linearly independent.]

(f) If $W$ is the subspace of $\mathbb{R}^4$ of all solutions to $x_1+2x_2+x_3-x_4=0$, then $\dim W=\boxed{\phantom{xx}}$, $\dim W^{\perp}=\boxed{\phantom{xx}}$.

**Solution.**

(a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

(b) $\hat{\boldsymbol{x}}$ is a least squares solution of $A\boldsymbol{x}=\boldsymbol{b}$ if and only if $A^TA\boldsymbol{x}=A^T\boldsymbol{b}$.

(c) $\operatorname{col}(A)$ is the orthogonal complement of $\operatorname{null}(A^T)$. $\operatorname{null}(A)$ is the orthogonal complement of $\operatorname{row}(A)$.

(d) The linear system $A\boldsymbol{x}=\boldsymbol{b}$ is consistent if and only if $\boldsymbol{b}$ is orthogonal to $\operatorname{null}(A^T)$.

(e) The projection matrix for orthogonally projecting onto $\operatorname{col}(A)$ is $P=A(A^TA)^{-1}A^T$.

(f) $\dim W=4-1=3$ and $\dim W^{\perp}=4-3=1$.