

# Preparing for Midterm #2

MATH 316 — Linear Algebra II  
Wednesday, April 8

*Please print your name:*

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**Bonus challenge.** Let me know about any typos you spot in the posted solutions (or lecture sketches or any other posted material). Any typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

**Problem 1.** Before working on the problems below:

- (a) Complete all homework! (In particular, Homework Sets 6, 7 and 8.)
- (b) Review the online practice problems (these are a particularly relevant subset of the homework problems, which will definitely show up on the exam).

**Problem 2.** Solve the initial value problem  $\mathbf{y}' = \begin{bmatrix} 4 & -8 \\ -1 & 6 \end{bmatrix} \mathbf{y}$ ,  $\mathbf{y}(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ .

(Use Homework Problem 8.2 to generate more practice problems of this kind.)

**Problem 3.**

- (a) Convert the third-order differential equation

$$y''' = 6y'' - 3y' - 10y, \quad y(0) = 1, \quad y'(0) = 2, \quad y''(0) = 3$$

to a system of first-order differential equations.

- (b) Solve the original differential equation by solving the system.

(Use Homework Problems 8.3, 8.4 to generate more practice problems of this kind.)

**Problem 4.**

- (a) What are the possible Jordan normal forms of a  $6 \times 6$  matrix with eigenvalues 7, 7, 3, 3, 3, 3?
- (b) How many different Jordan normal forms are there for a  $10 \times 10$  matrix with eigenvalues 8, 6, 6, 2, 2, 2, 1, 1, 1, 1?

(Use Homework Problems 8.5, 8.6 to generate more practice problems of this kind.)

**Problem 5.** Consider the sequence  $a_n$  defined by  $a_{n+2} = 4a_{n+1} - a_n$  and  $a_0 = 1$ ,  $a_1 = 0$ .

- (a) Determine the next three terms.

(b) A Binet-like formula for  $a_n$  is  $a_n =$

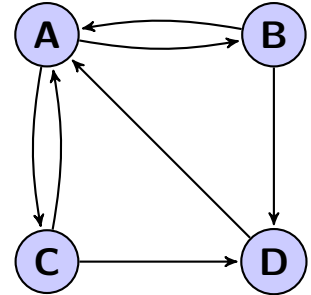
(c)  $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} =$

(Use Homework Problems 6.6, 7.1, 7.2, 7.3 to generate more practice problems of this kind.)

**Problem 6.** Suppose the internet consists of only the four webpages  $A, B, C, D$  which link to each other as indicated in the diagram.

Rank these webpages by computing their PageRank vector.

(Use Homework Problem 6.4 to generate more practice problems of this kind.)



**Problem 7.** Determine an orthogonal matrix  $P$  and a diagonal matrix  $D$  such that  $A = PDP^T$ .

(a) If  $A$  is the  $3 \times 3$  matrix for reflecting through the plane spanned by the vectors  $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ .

(b) If  $A$  is the  $3 \times 3$  matrix for reflecting through the plane spanned by the vectors  $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ .

(Use Homework Problem 7.6 to generate more practice problems of this kind.)

**Problem 8.**

(a) The eigenvalues of a  $5 \times 5$  matrix for orthogonally projecting onto a 3-dimensional subspace are .  
What are the eigenspaces of that matrix?

(b) Suppose  $A$  is the  $3 \times 3$  matrix of a reflection through a plane (containing the origin).

Then  $\det(A) =$  , and the eigenvalues of  $A$  are . What are the eigenspaces of  $A$ ?

(c) Precisely state the spectral theorem.

(d) If  $A$  is a reflection matrix, then  $A^{-1} =$  .

(e) If  $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ , then  $A^n =$   and  $e^{At} =$  .

(f) If  $N^4 = \mathbf{0}$ , then  $e^{Nt} =$  .