Problem 1. (4 points) Solve the initial value problem $\boldsymbol{y}^{\prime}=\left[\begin{array}{ll}0 & 1 \\ 2 & 1\end{array}\right] \boldsymbol{y}, \quad \boldsymbol{y}(0)=\left[\begin{array}{l}1 \\ 3\end{array}\right]$ on the extra sheet.

The solution is $\boldsymbol{y}(t)=$


Make sure to check your answer by plugging into the differential equation! There will be no partial credit and you have plenty of time.

Problem 2. ( $\mathbf{1}+\mathbf{3}+\mathbf{1}$ points) Consider the sequence $a_{n}$ defined by $a_{n+2}=2 a_{n+1}+3 a_{n}$ and $a_{0}=1, a_{1}=7$.
(a) The next two terms are $a_{2}=\square$ and $a_{3}=\square$
(b) A Binet-like formula for $a_{n}$ is $a_{n}=$
(c) $\lim _{n \rightarrow \infty} \frac{a_{n+1}}{a_{n}}=\square$

Again, work on the extra sheet and be sure to check your answer to (b) by comparing with the values in (a).

Problem 3. ( $\mathbf{1}+\mathbf{1}+\mathbf{2}+\mathbf{2}$ points) Fill in the blanks.
(a) If $A=\left[\begin{array}{cc}3 & 0 \\ 0 & -1\end{array}\right]$, then $e^{A t}=$ $\square$
(b) An example of a $2 \times 2$ matrix that is not diagonalizable is $\square$
(c) If $A$ has eigenvalue 3 , then $A^{2}$ has eigenvalue $\square$ $4 A$ eigenvalue $\square$ and $A^{T}$ eigenvalue $\square$
(d) How many different Jordan normal forms are there in the following cases?

- A $5 \times 5$ matrix with eigenvalues $1,1,2,2,2$ ? $\square$
- A $9 \times 9$ matrix with eigenvalues $1,1,2,2,2,4,4,4,4$ ? $\square$

