## Midterm #2

Please print your name:

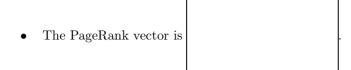
No notes, calculators or tools of any kind are permitted. There are 31 points in total. You need to show work to receive full credit.

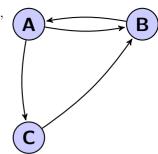
## Good luck!

**Problem 1.** (8 points) Solve the initial value problem  $\mathbf{y}' = \begin{bmatrix} 1 & 3 \\ -1 & 5 \end{bmatrix} \mathbf{y}, \quad \mathbf{y}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$ 

**Problem 2.** (6 points) Suppose the internet consists of only the three webpages A, B, C which link to each other as indicated in the diagram.

Rank these webpages by computing their PageRank vector:





The ranking of the websites is

The runking of the websites is	

Problem 3. (5 points) Fill in the blanks.

(a) Let A be the  $4 \times 4$  matrix for orthogonally projecting onto a 2-dimensional subspace of  $\mathbb{R}^4$ .

Then det(A) =and the eigenvalues (indicate if repeated) of A are

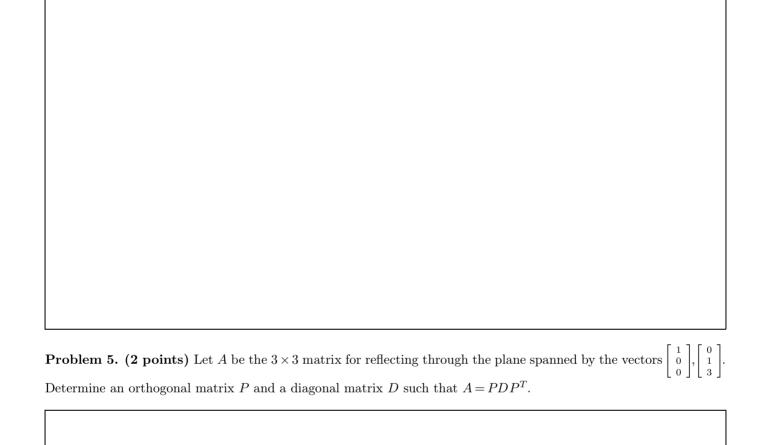
(b) If A is a projection matrix, then  $A^{2020} =$ 

(c) If A is a reflection matrix, then  $A^{2020} =$ 

and  $A^T$  eigenvalue (d) If A has eigenvalue 2, then  $A^3$  has eigenvalue 3A eigenvalue

(e) If  $A = \begin{bmatrix} -2 & 0 \\ 0 & 4 \end{bmatrix}$ , then  $A^n =$ and  $e^{At} =$  **Problem 4.** (1+4+1 points) Consider the sequence  $a_n$  defined by  $a_{n+2} = a_{n+1} + 2a_n$  and  $a_0 = 1$ ,  $a_1 = 8$ .

- (a) The next two terms are  $a_2 = \boxed{}$  and  $a_3 = \boxed{}$
- (b) A Binet-like formula for  $a_n$  is  $a_n = \begin{bmatrix} & & \\ & & \\ & & \\ & & \end{bmatrix}$ , and  $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \begin{bmatrix} & & \\ & & \\ & & \\ & & \end{bmatrix}$



Problem 6. (1+1+2 points) Fill in the blanks.

- (a) An example of a  $2\times 2$  matrix with eigenvalue  $\lambda=5$  that is not diagonalizable is
- (b) If  $N^3 = \mathbf{0}$ , then  $e^{Nt} =$
- (c) How many different Jordan normal forms are there in the following cases?
  - A  $4 \times 4$  matrix with eigenvalues 2, 5, 5, 5?
  - A  $8 \times 8$  matrix with eigenvalues 1, 1, 2, 2, 4, 4, 4, 4?

(extra scratch paper)