## Midterm #1

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 30 points in total. You need to show work to receive full credit.

## Good luck!

## Problem 1. (6 points)

- (a) Find the least squares solution to  $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} -1 \\ 0 \\ 2 \\ 5 \end{bmatrix}.$
- (b) Determine the least squares line for the data points (2, -1), (1, 0), (1, 2), (-1, 5).

## Problem 2. (9 points)

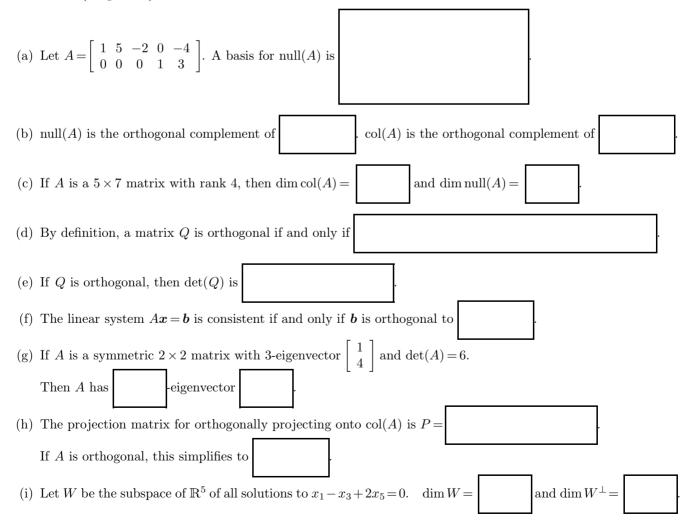
- (a) Using Gram–Schmidt, obtain an orthonormal basis for  $W = \operatorname{span} \left\{ \begin{bmatrix} 1\\1\\0 \end{bmatrix}, \begin{bmatrix} 3\\1\\-1 \end{bmatrix} \right\}$ . (b) Determine the orthogonal projection of  $\begin{bmatrix} 2\\0\\1 \end{bmatrix}$  onto W. (c) Determine the QR decomposition of the matrix  $\begin{bmatrix} 1&3\\1&1\\0&-1 \end{bmatrix}$ .
- (d) Determine a basis for the orthogonal complement  $W^{\perp}$ .

**Problem 3. (3 points)** We want to find values for the parameters a, b, c such that  $z = ax + bx^2 + c\ln(y)$  best fits some given points  $(x_1, y_1, z_1), (x_2, y_2, z_2), \dots$  Set up a linear system such that  $[a, b, c]^T$  is a least squares solution.

**Problem 4. (2 points)** Write down a precise definition of what it means for vectors  $v_1, v_2, ..., v_m \in \mathbb{R}^n$  to be linearly independent.

Vectors  $v_1, v_2, ..., v_m \in \mathbb{R}^n$  are linearly independent if and only if ...

Problem 5. (10 points) Fill in the blanks.



(extra scratch paper)