Midterm #2

MATH 316 — Linear Algebra II
Wednesday, Mar 21

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 29 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (8 points) Consider $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$.

(a) Determine the SVD of A.

(b) Determine the pseudoinverse of A.

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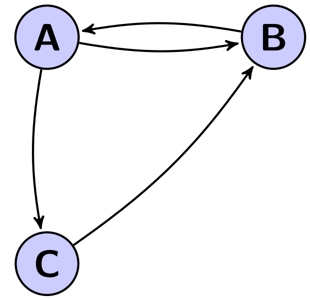
Problem 2. (6 points) Suppose the internet consists of only the three webpages A, B, C which link to each other as indicated in the diagram.



Rank these webpages by computing their PageRank vector.

Problem 3. (3 points) Let A be the 3×3 matrix for reflecting through the plane spanned by the vectors $\begin{bmatrix}2\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$.

Determine an orthogonal matrix P and a diagonal matrix D such that $A = PDP^T$.

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Problem 4. (2 points) Write down a precise definition of what it means for vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m \in \mathbb{R}^n$ to be linearly independent.

Vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m \in \mathbb{R}^n$ are linearly independent if and only if ...

Problem 5. (10 points) Fill in the blanks.

(a) Let A be the 3×3 matrix for an orthogonal projection onto a 2-dimensional subspace.

Then $\det(A) =$ ______, and the eigenvalues of A are ______.

(b) If A is $n \times n$, then the product of its singular values equals ______.

(c) The pseudoinverse of $A = \begin{bmatrix} 4 & 0 \\ 0 & -2 \\ 0 & 0 \end{bmatrix}$ is $A^+ =$ ______.

(d) The 2×2 rotation matrix by angle θ is ______.

(e) If A has full column rank, then its pseudoinverse is given by the formula $A^+ =$ ______.

(f) Suppose the linear system $A\boldsymbol{x} = \boldsymbol{b}$ has infinitely many solutions $\boldsymbol{x}$.

Which of these solutions is produced by $A^+\boldsymbol{b}$? ______

(g) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$, then $A^n =$ ______.

(h) If A is a projection matrix, then $A^{2018} =$ ______.

(i) If A is a reflection matrix, then $A^{2018} =$ ______.

(j) If A has SVD $A = U\Sigma V^T$, then A^T has SVD ______.

(extra scratch paper)