Midterm #1

MATH 316 — Linear Algebra II
Friday, Feb 16

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 28 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (6 points)

(a) Find the least squares solution to $\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 3 \\ 2 \\ 0 \\ -1 \end{bmatrix}$.

(b) Determine the least squares line for the data points $(0,3),(1,2),(1,0),(2,-1)$.

Armin Straub
straub@southalabama.edu

Problem 2. (9 points)

(a) Using Gram–Schmidt, obtain an orthonormal basis for $W = \operatorname{span}\left\{ \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} \right\}$.

(b) Determine the orthogonal projection of $\begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$ onto W.

(c) Determine the QR decomposition of the matrix $\begin{bmatrix} 2 & 4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$.

(d) Determine a basis for the orthogonal complement $W^{\perp}$.

Problem 3. (3 points) We want to find values for the parameters a, b, c such that $z = a + bx + cy$ best fits some given points $(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots$ Set up a linear system such that $[a, b, c]^T$ is a least squares solution.

Problem 4. (2 points) Write down a precise definition of what it means for vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m \in \mathbb{R}^n$ to be linearly independent.

Vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m \in \mathbb{R}^n$ are linearly independent if and only if ...

Problem 5. (8 points) Fill in the blanks.

(a) If A is a 7×5 matrix with rank 3, then $\dim \operatorname{col}(A) =$ ☐, $\dim \operatorname{row}(A) =$ ☐, $\dim \operatorname{null}(A) =$ ☐.

(b) If B is a 5×3 matrix, then $\operatorname{null}(B)$ is a subspace of ☐ and $\operatorname{col}(B)$ is a subspace of ☐.

(c) $\hat{\boldsymbol{x}}$ is a least squares solution of $A\boldsymbol{x} = \boldsymbol{b}$ if and only if ☐.

(d) $\operatorname{col}(A)$ is the orthogonal complement of ☐. $\operatorname{null}(A)$ is the orthogonal complement of ☐.

(e) The linear system $A\boldsymbol{x} = \boldsymbol{b}$ is consistent if and only if $\boldsymbol{b}$ is orthogonal to ☐.

(f) The projection matrix for orthogonally projecting onto $\operatorname{col}(A)$ is $P =$ ☐.

(g) If P is a projection matrix, then $P^2 =$ ☐.

(h) If W is the space of all solutions to $x_1 + 2x_2 + x_3 - x_4 = 0$, then $\dim W =$ ☐ and $\dim W^{\perp} =$ ☐.

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(extra scratch paper)