

Quiz #1

Please print your name:

Problem 1. Consider the following system of linear equations:

$$\begin{aligned}x_1 + 3x_2 - x_3 &= 5 \\x_1 + 4x_2 + 4x_3 &= -3 \\2x_1 + 7x_2 + x_3 &= 6\end{aligned}$$

- (a) Write down the augmented matrix for the system.
- (b) Using Gaussian elimination, determine the RREF. (Make sure to record all your row operations!)
- (c) From the RREF, read off the solution to the linear system.

Solution.

(a) The augmented matrix is $\left[\begin{array}{ccc|c} 1 & 3 & -1 & 5 \\ 1 & 4 & 4 & -3 \\ 2 & 7 & 1 & 6 \end{array} \right]$.

(b) $\left[\begin{array}{ccc|c} 1 & 3 & -1 & 5 \\ 1 & 4 & 4 & -3 \\ 2 & 7 & 1 & 6 \end{array} \right] \xrightarrow[\sim]{\substack{R_2 - R_1 \Rightarrow R_2 \\ R_3 - 2R_1 \Rightarrow R_3}} \left[\begin{array}{ccc|c} 1 & 3 & -1 & 5 \\ 0 & 1 & 5 & -8 \\ 0 & 1 & 3 & -4 \end{array} \right] \xrightarrow[\sim]{R_3 - R_2 \Rightarrow R_3} \left[\begin{array}{ccc|c} 1 & 3 & -1 & 5 \\ 0 & 1 & 5 & -8 \\ 0 & 0 & -2 & 4 \end{array} \right]$

$\xrightarrow[\sim]{-\frac{1}{2}R_3 \Rightarrow R_3} \left[\begin{array}{ccc|c} 1 & 3 & -1 & 5 \\ 0 & 1 & 5 & -8 \\ 0 & 0 & 1 & -2 \end{array} \right] \xrightarrow[\sim]{\substack{R_1 + R_3 \Rightarrow R_1 \\ R_2 - 5R_3 \Rightarrow R_2}} \left[\begin{array}{ccc|c} 1 & 3 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array} \right] \xrightarrow[\sim]{R_1 - 3R_2 \Rightarrow R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array} \right]$

The final matrix is the row-reduced echelon form.

(c) The system has the unique solution $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ -2 \end{bmatrix}$.

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