# Preparing for the Final

**MATH 237 — Linear Algebra I**
**Fall 2016**

***Please print your name:***

Our final exam will be comprehensive, with a focus on the material learned later in the semester.

(Note that lots of the things we learned more recently require us to know earlier material anyway.)

A good way to prepare yourself is to study the following:

- redo the practice problems for Midterm 1 and Midterm 2,
- do the problems below,
- retake the midterm exams and quizzes,
- go through the lecture sketches.

  Make sure that you can briefly but precisely define our important notions (linear independence, basis, rank, dimension, ...). These are in bold face in the lecture sketches. The sketches also contain lots of (computationally pleasant) problems with solutions.

# I Computational part

**Problem 1.** Let $A = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

(a) Find the eigenvalues and bases for the eigenspaces of $A$.

(b) If possible, diagonalize $A$. That is, determine matrices $P$ and $D$ such that $A = PDP^{-1}$.

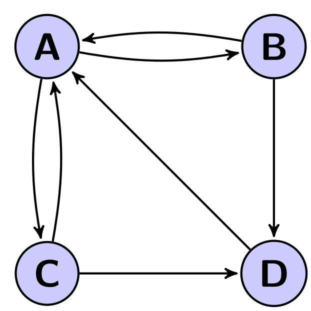
**Problem 2.** Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -5 & 0 \\ 1 & 8 & 1 \end{bmatrix}$.

(a) Find the eigenvalues and bases for the eigenspaces of $A$.

(b) If possible, diagonalize $A$. That is, determine matrices $P$ and $D$ such that $A = PDP^{-1}$.

**Problem 3.** Suppose the internet consists of only the four webpages $A, B, C, D$ which link to each other as indicated in the diagram.

Rank these webpages by computing their PageRank vector.




**Problem 4.** Find a basis and the dimension of $W = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.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# II Short answer part

**Problem 5.** Suppose $A$ is a $5 \times 5$ matrix with eigenvalue 0.

(a) What can you say about $\operatorname{rank}(A)$?

(b) What can you say about $\operatorname{rank}(A)$ if the multiplicity of 0 is 1?

(c) What can you say about $\operatorname{rank}(A)$ if the multiplicity of 0 is 2?

**Problem 6.** Produce a $2 \times 2$ matrix which has 1-eigenvector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and 3-eigenvector $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$. Are there others?

**Problem 7.**

(a) What does it mean for two matrices $A, B$ to be similar?

(b) Show that similar matrices have the same characteristic polynomial.

(c) Is it true that similar matrices have the same eigenvalues? Is it true that similar matrices have the same eigenvectors? Explain.

**Problem 8.** Let $A$ be a $n \times n$ matrix. List at least five other statements which are equivalent to the statement "$A$ is invertible".

**Problem 9.** Determine whether each of the following "laws" is true for all (invertible) $n \times n$ matrices $A, B$.

(a) $(AB)^T = A^T B^T$

(b) $(AB)^T = B^T A^T$

(c) $(AB)^{-1} = A^{-1} B^{-1}$

(d) $(AB)^{-1} = B^{-1} A^{-1}$

**Problem 10.** Describe $\operatorname{col}(A), \operatorname{row}(A), \operatorname{null}(A)$ if $A$ is an invertible $n \times n$ matrix.

**Problem 11.** You overhear a conversation during which someone explains that "matrix inverses are amazing because they allow us to solve any linear system $A\boldsymbol{x} = \boldsymbol{b}$ by simply computing $\boldsymbol{x} = A^{-1}\boldsymbol{b}$". What is your take on this statement?

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