Quiz #10

MATH 237 — Linear Algebra I
Tuesday, Nov 24

Please print your name:

Problem 1. Find the eigenvectors and eigenvalues of $A = \begin{bmatrix} 0 & -2 \\ -4 & 2 \end{bmatrix}$.

Solution.

- The characteristic polynomial is:

  $\det(A - \lambda I) = \begin{vmatrix} -\lambda & -2 \\ -4 & 2-\lambda \end{vmatrix} = -\lambda(2-\lambda) - 8 = \lambda^2 - 2\lambda - 8 = (\lambda - 4)(\lambda + 2)$

- A has eigenvalues $\lambda_1 = 4$ and $\lambda_2 = -2$.

- $\lambda_1 = 4$:

  $(A - \lambda_1 I)\boldsymbol{x} = \begin{bmatrix} -4 & -2 \\ -4 & -2 \end{bmatrix}\boldsymbol{x} = \mathbf{0} \implies \boldsymbol{x}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ (or any multiple)

- $\lambda_2 = -2$:

  $(A - \lambda_2 I)\boldsymbol{x} = \begin{bmatrix} 2 & -2 \\ -4 & 4 \end{bmatrix}\boldsymbol{x} = \mathbf{0} \implies \boldsymbol{x}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (or any multiple)

- In summary, A has eigenvalues $4, -2$ with corresponding eigenvectors $\begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

- We check our answer:

  $\begin{bmatrix} 0 & -2 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ -8 \end{bmatrix} \overset{\checkmark}{=} 4\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

  $\begin{bmatrix} 0 & -2 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix} \overset{\checkmark}{=} -2\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

□

Armin Straub
straub@southalabama.edu