Quiz #8

MATH 237 — Linear Algebra I
Thursday, Oct 29

Please print your name:

Problem 1. Find a basis for col(A), row(A) and null(A) for $A=\begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & 4 & 2 & 6 & 12 \\ 3 & 6 & 1 & 7 & 14 \end{bmatrix}$. (Make sure to show your work!)

Solution.

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & 4 & 2 & 6 & 12 \\ 3 & 6 & 1 & 7 & 14 \end{bmatrix} \overset{\substack{R_2-2R_1\Rightarrow R_2 \\ R_3-3R_1\Rightarrow R_3}}{\rightsquigarrow} \begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 0 & 4 & 4 & 8 \\ 0 & 0 & 4 & 4 & 8 \end{bmatrix} \overset{\substack{R_3-R_2\Rightarrow R_3 \\ \frac{1}{2}R_2\Rightarrow R_2}}{\rightsquigarrow} \begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \overset{R_1+R_2\Rightarrow R_1}{\rightsquigarrow} \begin{bmatrix} 1 & 2 & 0 & 2 & 4 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The first and third column contain a pivot. Hence, a basis for col(A) is given by $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$.

A basis for row(A) is given by $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}$.

The general solution to $A\boldsymbol{x}=\boldsymbol{0}$ is $\boldsymbol{x}=\begin{bmatrix} -2s_1-2s_2-4s_3 \\ s_1 \\ -s_2-2s_3 \\ s_2 \\ s_3 \end{bmatrix} = s_1\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s_2\begin{bmatrix} -2 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + s_3\begin{bmatrix} -4 \\ 0 \\ -2 \\ 0 \\ 1 \end{bmatrix}$.

Hence, a basis for null(A) is $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ -2 \\ 0 \\ 1 \end{bmatrix}$. □

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