Preparing for Midterm #2

MATH 237 — Linear Algebra I
Thursday, Nov 12

Please print your name:

Computational part

Problem 1. Evaluate the following determinants. [Real computations only necessary for the last two.]

(a) $\begin{vmatrix} 1 & 1 & 4 \\ 2 & 2 & 5 \\ 3 & 3 & 6 \end{vmatrix}$

(b) $\begin{vmatrix} 1 & 1 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 6 \end{vmatrix}$

(c) $\begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$

(d) $\begin{vmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 0 \\ -1 & -2 & 0 & 0 \\ 0 & 2 & 5 & 0 \end{vmatrix}$

(e) $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 3 \\ 3 & 2 & 1 \end{vmatrix}$

(f) $\begin{vmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{vmatrix}$

Problem 2. Find a basis for $\operatorname{col}(A), \operatorname{row}(A), \operatorname{null}(A)$ with

(a) $A = \begin{bmatrix} 1 & 2 & 1 & 1 & 5 \\ -1 & -2 & -1 & -1 & -3 \\ 2 & 4 & 0 & -6 & 7 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(c) $A = [\ 1 \ \ 2 \ \ 3\ ]$

Problem 3.

(a) Is $W = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} : \ a - b = c, \ a - d = e \right\}$ a vector space? If yes, find a basis.

(b) Is $W = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ a vector space? If yes, find a basis.

(c) Is $W = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ a vector space? If yes, find a basis.

(d) Is $W = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ a vector space? If yes, find a basis.

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Problem 4. Consider $H = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$.

(a) Give a basis for H. What is the dimension of H?

(b) Determine whether the vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is in H. What about the vector $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$?

(c) Extend the basis of H to a basis of $\mathbb{R}^3$.

Problem 5. Is it true that $\operatorname{span}\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\} = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}$?

Short answer part

Problem 6. Let A be a 5×4 matrix. Suppose that the linear system $A\boldsymbol{x} = \boldsymbol{b}$ has the solution set

$$\left\{ \begin{bmatrix} 1 - c + d \\ c \\ 3 - 2d \\ d \end{bmatrix} : c, d \text{ in } \mathbb{R} \right\}.$$

(a) Give a basis for the null space of A.

(b) What is the rank of A?

Problem 7. In each case, write down a precise definition or answer.

(a) What is a vector space?

(b) What is the rank of a matrix?

(c) What does it mean for vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m$ from a vector space to be linearly independent?

(d) List the elementary row operations.

(e) What does it mean for vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m$ to be a basis for a vector space V?

Problem 8. Let A be a $n \times n$ matrix. List at least five other statements which are equivalent to the statement "A is invertible".

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Problem 9.

(a) Suppose V and W are subspaces of $\mathbb{R}^n$, and that $\boldsymbol{v}_1, \boldsymbol{v}_2$ is a basis for V, and $\boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3$ is a basis for W. What can you say about $\dim U$ with $U = \operatorname{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3\}$?

(b) Let A be a 4×3 matrix, whose row space has dimension 2. What is the dimension of $\operatorname{null}(A)$?

(c) Let A be a 3×3 matrix, whose column space has dimension 3. If $\boldsymbol{b}$ is a vector in $\mathbb{R}^3$, what can you say about the number of solutions to the equation $A\boldsymbol{x} = \boldsymbol{b}$?

(d) Let A be a 3×3 matrix, whose column space has dimension 2. What can you say about $\det(A)$?

Problem 10. True or false?

(a) Every vector space has a basis.

(b) The zero vector can never be a basis vector.

(c) Every set of linearly independent vectors in V can be extended to a basis of V.

(d) $\operatorname{col}(A)$ and $\operatorname{row}(A)$ always have the same dimension.

(e) If B is the RREF of A, then we always have $\operatorname{col}(A) = \operatorname{col}(B)$.

(f) If B is the RREF of A, then we always have $\operatorname{row}(A) = \operatorname{row}(B)$.

(g) If a subspace V of $\mathbb{R}^3$ contains three linearly independent vectors, then always $V = \mathbb{R}^3$.

(h) There are matrices A such that $\operatorname{null}(A)$ is the empty set.

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