Preparing for Midterm #1

MATH 237 — Linear Algebra I
Thursday, Oct 1

Please print your name:

Problem 1. Compute the following, or state why it is not possible to do so:

(a) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 3 & 4 \end{bmatrix}^T$

(b) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 3 & 4 \end{bmatrix}^{-1}$

(c) $\begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}^{-1}$

(d) $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}^{-1}$

(e) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}$

(f) $\begin{bmatrix} 3 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$

Problem 2. Let A be a $p \times q$ matrix and B be an $r \times s$ matrix. Under which condition is $A^T B$ defined?

Problem 3. Decide whether the following statements are true or false.

(a) If A is invertible then the system $A\boldsymbol{x} = \boldsymbol{b}$ always has the same number of solutions.

(b) The homogeneous system $A\boldsymbol{x} = \boldsymbol{0}$ is always consistent.

(c) In order for A to be invertible, the matrix A has to be square (that is, of shape $n \times n$).

(d) If A is a 4×3 matrix with 2 pivot columns, then the columns of A are linearly independent.

(e) If A is invertible then the columns of A are linearly independent.

(f) $\boldsymbol{b}$ is in the span of the columns of A if and only if the system $A\boldsymbol{x} = \boldsymbol{b}$ is consistent.

(g) Every matrix can be reduced to echelon form by a sequence of elementary row operations.

(h) The row-reduced echelon form of a matrix is unique.

Problem 4. We are solving a linear system with 4 equations and 5 unknowns. Which of the following are possible?

(a) The system has no solution.

(b) The system has a unique solution.

(c) The system has infinitely many solutions.

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Problem 6. For which values of a is the matrix $\begin{bmatrix} 3 & a-6 \\ 3a & -a+6 \end{bmatrix}$ invertible?

Problem 7. Consider the vectors

$$\boldsymbol{w} = \begin{bmatrix} 2 \\ -4 \\ -1 \\ 1 \end{bmatrix}, \quad \boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

(a) Is $\boldsymbol{w}$ in $\operatorname{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$? If so, write $\boldsymbol{w}$ as a linear combination of $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$.

(b) Determine whether the vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ are linearly independent.

(c) Is $\boldsymbol{v}_3$ in $\operatorname{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2\}$?

Problem 8. Consider the vectors

$$\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}, \quad \boldsymbol{v}_3 = \begin{bmatrix} 1 \\ 0 \\ h \end{bmatrix}.$$

(a) For which value(s) of h is $\boldsymbol{v}_3$ a linear combination of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$?

(b) For which value(s) of h are $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ linearly independent?

Problem 9. Consider $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

(a) Determine A^{-1}.

(b) Using (a), solve $A\boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

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Problem 10. Consider $B = \begin{bmatrix} 1 & 2 & 6 & 5 & -5 & 0 \\ 2 & 4 & 14 & 12 & -12 & -2 \\ 1 & 2 & 4 & 3 & -2 & 6 \end{bmatrix}$.

(a) Determine the row-reduced echelon form of B.

(b) Use your result in (a) to find the general solution of the linear system:

$$\begin{array}{rrrrrcr} x_1 & +2x_2 & +6x_3 & +5x_4 & -5x_5 & = & 0 \\ 2x_1 & +4x_2 & +14x_3 & +12x_4 & -12x_5 & = & -2 \\ x_1 & +2x_2 & +4x_3 & +3x_4 & -2x_5 & = & 6 \end{array}$$

(c) What is the general solution to the associated homogeneous linear system?

(d) Write down, in vector form, the general solution to $\begin{bmatrix} 1 & 2 & 6 & 5 & -5 \\ 2 & 4 & 14 & 12 & -12 \\ 1 & 2 & 4 & 3 & -2 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 0 \\ -2 \\ 6 \end{bmatrix}$. Also decompose the solution into a particular solution and the solutions to the homogeneous system.

(e) What is the general solution to $\begin{bmatrix} 1 & 2 & 6 & 5 & -5 \\ 2 & 4 & 14 & 12 & -12 \\ 1 & 2 & 4 & 3 & -2 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$? (Note that the RHS is also a column of the matrix!)

(f) Are the columns of $\begin{bmatrix} 1 & 2 & 6 & 5 & -5 \\ 2 & 4 & 14 & 12 & -12 \\ 1 & 2 & 4 & 3 & -2 \end{bmatrix}$ linearly independent? If not, write down a non-trivial linear combination of the columns, which produces $\mathbf{0}$.

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