Homework #7

MATH 237 — Linear Algebra I
due Thursday, Oct 29, in class

Please print your name:

Problem 1. Let $A = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 2 & 4 & -1 & 3 \\ 3 & 6 & 2 & 22 \end{bmatrix}$.

(a) Find a basis for $\operatorname{col}(A)$, $\operatorname{row}(A)$ and $\operatorname{null}(A)$.

(b) Let $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_r$ be your basis for $\operatorname{row}(A)$, and let $\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_s$ be your basis for $\operatorname{null}(A)$. Find a basis for $W = \operatorname{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_r, \boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_s\}$. Conclude that $\dim W = 4$, and hence $W = \mathbb{R}^4$.

[*Comment:* What you observe here is always the case! Let A be a $m \times n$ matrix. Note that $\operatorname{row}(A)$ and $\operatorname{null}(A)$ are both subspaces of $\mathbb{R}^n$. Further, it follows from what we learned in class that $\dim \operatorname{row}(A) + \dim \operatorname{null}(A) = n$. What you observed here is that, in fact, taking the sum of $\operatorname{row}(A)$ and $\operatorname{null}(A)$ gives you all of $\mathbb{R}^n$.
"Not only the dimensions add up to the full dimension, but the spaces add up to the full space!"]

Problem 2. Let A be a $m \times n$ matrix.

(a) For each of $\operatorname{col}(A)$, $\operatorname{row}(A)$ and $\operatorname{null}(A)$, state which space $\mathbb{R}^{??}$ they are a subspace of.

(b) Why is $\dim \operatorname{row}(A) + \dim \operatorname{null}(A) = n$?

(c) Suppose that the columns of A are independent. What can you say about the dimensions of $\operatorname{col}(A)$, $\operatorname{row}(A)$ and $\operatorname{null}(A)$?

(d) Suppose that A has **rank** 2 (that is, an echelon form of A has exactly 2 pivots). What can you say about the dimensions of $\operatorname{col}(A)$, $\operatorname{row}(A)$ and $\operatorname{null}(A)$?

Armin Straub
straub@southalabama.edu