ANSWERS TO SELECTED EXERCISES

Chapter 1

Section 1.1

1. Only $(-3, -3)$ lies on line.

3. Only $(-2, 5)$ lies on both lines.

5. None satisfies the linear system.

7. Only (b), (c), and (d) are solutions to the linear system.

9. $x_1 = 3, x_2 = -1$.

11. $x_1 = s_1, x_2 = \frac{1}{2} + \frac{5}{2}s_1$.

13. $x_1 = -\frac{8}{41}, x_2 = -\frac{5}{41}$.

15. Echelon form; x_1, x_2 leading variables, no free variables.

17. Echelon form; x_1, x_3 leading variables, x_2 a free variable.

19. Not in echelon form.

21. Echelon form; x_1, x_3 leading variables, x_2, x_4 free variables.

23. $x_1 = -\frac{19}{5}, x_2 = 5$.

25. $x_1 = -\frac{2}{3} + \frac{4}{3}s_1, x_2 = s_1$.

27. $x_1 = 10 - \frac{1}{2}s_1, x_2 = -2 + \frac{1}{2}s_1, x_3 = s_1, x_4 = 5$.

29. $x_1 = \frac{5}{6} + \frac{1}{2}s_1 + \frac{1}{3}s_2, x_2 = s_1, x_3 = \frac{4}{3} + \frac{1}{3}s_2, x_4 = s_2$.

31. Reverse order of equations; $x_1 = \frac{13}{15}, x_2 = -\frac{4}{5}$.

33. Reverse order of equations; $x_1 = -1 + \frac{7}{2}s_1 - \frac{19}{2}s_2$, $x_2 = -\frac{1}{2}s_1 + \frac{5}{2}s_2, x_3 = s_1, x_4 = s_2$.

35. $k \neq -\frac{15}{2}$.

37. $h = 2, k \neq -1$.

39. 9 variables.

41. 7 leading variables.

43. For example,

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 0 \\ x_3 &= 0 \end{aligned}$$

45. For example,

$$\begin{aligned} x_1 + x_2 \phantom{{}- x_3} &= 0 \\ x_1 + x_2 - x_3 &= 0 \\ x_3 &= 0 \\ x_1 + x_2 + x_3 &= 0 \end{aligned}$$

47. On Monday, I bought 3 apples and 4 oranges and spent \$0.55. On Tuesday I bought 6 oranges and spent \$0.60. How much does each apple and orange cost?

Answer: apples cost 5 cents each and oranges cost 10 cents each.

49. For example,

$$\begin{aligned} x_1 - x_2 \phantom{{}- x_3} &= -3 \\ 3x_1 \phantom{{}- x_2} - x_3 &= 4 \end{aligned}$$

51. False

53. False

55. True

57. False

59. True

61. 196.875 liters of 18% solution, 103.125 liters of 50% solution.

63. 298 adults and 87 children.

65. $a_1 = \frac{14}{5}$ and $a_2 = \frac{11}{5}$

67. $a = \frac{5}{9}$ and $b = -\frac{160}{9}$

69. The published values from the United States Mint are $q = 0.955$ in and $n = 0.835$ in.

71. $x_1 = 12, x_2 = 5$

73. $x_1 = \frac{33}{8}s_1, x_2 = \frac{9}{4}s_1 - \frac{23}{11}, x_3 = s_1 - \frac{5}{33}$

75. $x_1 = \frac{47}{8}s_1, x_2 = -2s_1 + \frac{69}{47}, x_3 = \frac{7}{4}s_1 + \frac{565}{141}, x_4 = s_1 + \frac{202}{141}$

Section 1.2

1.
$$\begin{aligned} 4x_1 + 2x_2 - x_3 &= 2 \\ -x_1 \phantom{{}+ 2x_2} + 5x_3 &= 7 \end{aligned}$$

3.
$$\begin{aligned} 12x_2 - 3x_3 - 9x_4 &= 17 \\ -12x_1 + 5x_2 - 3x_3 + 11x_4 &= 0 \\ 6x_1 + 8x_2 + 2x_3 + 10x_4 &= -8 \\ 17x_1 \phantom{{}+ 8x_2 + 2x_3} + 13x_4 &= -1 \end{aligned}$$

5. Echelon form.

7. Not in echelon form.

9. Echelon form.

11. $-2R_1 \Rightarrow R_1$

13. $-2R_2 + R_3 \Rightarrow R_3$

15. $R_1 \Leftrightarrow R_2$, $\begin{bmatrix} -1 & 4 & 3 \\ 3 & 7 & -2 \\ 5 & 0 & -3 \end{bmatrix}$

17. $2R_1 \Rightarrow R_1$, $\begin{bmatrix} 0 & 6 & -2 & 4 \\ -1 & -9 & 4 & 1 \\ 5 & 0 & 7 & 2 \end{bmatrix}$

19. $\begin{bmatrix} 2 & 1 & 1 \\ -4 & -1 & 3 \end{bmatrix}$; $x_1 = -2, x_2 = 5$.

21. $\begin{bmatrix} -2 & 5 & -10 & 4 \\ 1 & -2 & 3 & -1 \\ 7 & -17 & 34 & -16 \end{bmatrix}$; $x_1 =- 12, x_2 =- 10, x_3 =- 3$.

23. $\begin{bmatrix} 2 & 2 & -1 & 8 \\ -1 & -1 & 0 & -3 \\ 3 & 3 & 1 & 7 \end{bmatrix}$; $x_1 = 3 - s_1, x_2 = s_1, x_3 = -2$.

25. $\begin{bmatrix} 2 & 6 & -9 & -4 & 0 \\ -3 & -11 & 9 & -1 & 0 \\ 1 & 4 & -2 & 1 & 0 \end{bmatrix}$;

$x_1 = 35s_1, x_2 = -8s_1, x_3 = 2s_1, x_4 = s_1$.

27. $\begin{bmatrix} -2 & -5 & 0 \\ 1 & 3 & 1 \end{bmatrix}$; $x_1 = -5$, $x_2 = 2$.

29. $\begin{bmatrix} 2 & 1 & 0 & 2 \\ -1 & -1 & -1 & 1 \end{bmatrix}$; $x_1 = 3+s_1, x_2 = -4-2s_1, x_3 = s_1$.

31. $(1/5)R_1 \Rightarrow R_1$

33. $R_1 \Leftrightarrow R_3$

35. $5R_2 + R_6 \Rightarrow R_6$

37. An example: $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

39. An example: $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

41. An example:

$$\begin{aligned} x_1 \quad\quad\quad\quad &= 0 \\ x_2 \quad\quad &= 0 \\ x_3 + x_4 &= 0 \end{aligned}$$

43. True

45. False

47. False

49. True

51. Exactly one solution.

53. HINT: Show that the system must have at least one free variable.

55. HINT: Show that the system must have at least one free variable.

57. $f(x) = 2x^2 - 3x + 5$

59. $E(x) = -\frac{1}{10}x^2 + \frac{49}{5}x + 132$

61. $x_1 = -\frac{157}{181}, x_2 = \frac{20}{181}, x_3 = -\frac{58}{181}$

63. $x_1 = \frac{7}{9} - s_1, x_2 = -\frac{23}{9} - s_1, x_3 = -\frac{22}{27} + s_1, x_4 = s_1$.

65. No solutions.

67. $x_1 = \frac{46}{579}s_1, x_2 = -\frac{745}{579}s_1, x_3 = -\frac{2264}{579}s_1,$
$x_4 = -\frac{655}{386}s_1, x_5 = s_1$.

Section 1.3

1. $x_1 = 1, x_2 = 2$.

3. $x_1 = \frac{79}{49}, x_2 = \frac{22}{49}, x_3 = \frac{124}{49}$.

5. No partial pivot: $x_1 = -0.219, x_2 = 0.0425$
With partial pivot: $x_1 = -0.180, x_2 = 0.0424$.

7. No partial pivot: $x_1 = -0.407, x_2 = -0.757, x_3 = 0.0124$
With partial pivot: $x_1 = -0.392, x_2 = -0.755, x_3 = 0.0124$.

9.

n	x_1	x_2
0	0	0
1	−1.2	0.2
2	−1.12	0.56
3	−0.976	0.536

Exact solution: $x_1 = -1, x_2 = 0.5$.

11.

n	x_1	x_2	x_3
0	0	0	0
1	−1.3	2.3	2.6
2	−2.295	3.34	1.42
3	−2.156	3.185	0.805

Exact solution: $x_1 = -2, x_2 = 3, x_3 = 1$.

13.

n	x_1	x_2
0	0	0
1	−1.2	0.56
2	−0.976	0.4928
3	−1.0029	0.5009

Exact solution: $x_1 = -1, x_2 = 0.5$.

15.

n	x_1	x_2	x_3
0	0	0	0
1	−1.3	2.56	1.316
2	−2.013	3.0974	0.9584
3	−2.0042	2.9884	1.0038

Exact solution: $x_1 = -2, x_2 = 3, x_3 = 1$.

17. Not diagonally dominant. Not possible to reorder to obtain diagonal dominance.

19. Not diagonally dominant. Not possible to reorder to obtain diagonal dominance.

21. Jacobi iteration of given linear system:

n	x_1	x_2
0	0	0
1	−1	−1
2	−3	−3
3	−7	−7
4	−15	−15

Diagonally dominant system:

$$\begin{aligned} 2x_1 - x_2 &= 1 \\ x_1 - 2x_2 &= -1 \end{aligned}$$

Jacobi iteration of diagonally dominant system:

n	x_1	x_2
0	0	0
1	0.5	0.5
2	0.75	0.75
3	0.875	0.875
4	0.9375	0.9375

23. Jacobi iteration of given linear system:

n	x_1	x_2	x_3
0	0	0	0
1	−1	8	−0.3333
2	16.67	12.33	27
3	−111.3	−21.33	29.67
4	−192	624	2.778

Diagonally dominant system:

$$\begin{aligned} 5x_1 + x_2 - 2x_3 &= 8 \\ 2x_1 - 10x_2 + 3x_3 &= -1 \\ x_1 - 2x_2 + 5x_3 &= -1 \end{aligned}$$

Jacobi iteration of diagonally dominant system:

n	x_1	x_2	x_3
0	0	0	0
1	1.6	0.1	−0.2
2	1.5	0.36	−0.48
3	1.336	0.256	−0.356
4	1.406	0.2604	−0.3648

25. Gauss–Seidel iteration of given linear system:

n	x_1	x_2
0	0	0
1	−1	−3
2	−7	−15
3	−31	−63
4	−127	−255

Diagonally dominant system:

$$\begin{aligned} 2x_1 - x_2 &= 1 \\ x_1 - 2x_2 &= -1 \end{aligned}$$

Gauss–Seidel iteration of diagonally dominant system:

n	x_1	x_2
0	0	0
1	0.5	0.75
2	0.875	0.9375
3	0.9688	0.9844
4	0.9922	0.9961

27. Gauss–Seidel iteration of given linear system:

n	x_1	x_2	x_3
0	0	0	0
1	−1	13	43.67
2	−193.3	1062	3669
3	-1.622×10^4	8.844×10^4	3.056×10^5
4	-1.351×10^6	7.367×10^6	2.546×10^7

Diagonally dominant system:

$$\begin{aligned} 5x_1 + x_2 - 2x_3 &= 8 \\ 2x_1 - 10x_2 + 3x_3 &= -1 \\ x_1 - 2x_2 + 5x_3 &= -1 \end{aligned}$$

Gauss–Seidel iteration of diagonally dominant system:

n	x_1	x_2	x_3
0	0	0	0
1	1.6	0.42	−0.352
2	1.375	0.2694	−0.3673
3	1.399	0.2697	−0.3712
4	1.397	0.2679	−0.3723

29. $x_1 = -3$, $x_2 = 18$.

31. $x_1 = 27$, $x_2 = 52$.

Section 1.4

1. Minimum = 20 vehicles.

3. Minimum = 25 vehicles.

5. $H_2 + O \longrightarrow H_2O$

7. $4Fe + 3O_2 \longrightarrow 2Fe_2O_3$

9. $C_3H_8 + 5O_2 \longrightarrow 3CO_2 + 4H_2O$

11. $4KO_2 + 2CO_2 \longrightarrow 2K_2CO_3 + 3O_2$

13. $p = (0.19847)\, d^{1.5011}$

15. $p = (0.20120)\, d^{1.49835}$

17. $d = 0.045s^2$

19. $A = 1$, $B = -1$.

21. $A = -1$, $B = -1$, $C = 1$.

23. $x = 4$

25. $y = -2x^2 + 3x - 5$

27. $g(x) = -x^4 + 2x^3 + x^2 - 3x + 5$

29. $f(x) = \frac{2}{3}e^{-2x} - \frac{5}{3}e^x + xe^x$

31. LAI = 0.0001100 (USA) + 0.0000586 (Harris) + 0.0066823 (Computer)

Chapter 2

Section 2.1

1. $\begin{bmatrix} 7 \\ -3 \\ -5 \end{bmatrix}$ **3.** $\begin{bmatrix} -10 \\ -4 \\ 14 \end{bmatrix}$ **5.** $\begin{bmatrix} -5 \\ -4 \\ 4 \end{bmatrix}$

7. $\begin{aligned} 3x_1 - x_2 &= 8 \\ 2x_1 + 5x_2 &= 13 \end{aligned}$

9. $\begin{aligned} -6x_1 + 5x_2 &= 4 \\ 5x_1 - 3x_2 + 2x_3 &= 16 \end{aligned}$

11. $x_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 8 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} -4 \\ 5 \end{bmatrix} = \begin{bmatrix} -10 \\ 4 \end{bmatrix}$

13. $x_1 \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 6 \\ 10 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix}$

15. $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \end{bmatrix} + s_1 \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

17. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -9 \\ 0 \end{bmatrix} + s_1 \begin{bmatrix} 6 \\ 0 \\ 3 \\ 1 \end{bmatrix} + s_2 \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

19. $1\mathbf{u} + 0\mathbf{v} = \mathbf{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, 0\mathbf{u} + 1\mathbf{v} = \mathbf{v} = \begin{bmatrix} -1 \\ -4 \end{bmatrix},$

$1\mathbf{u} + 1\mathbf{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$

21. $1\mathbf{u} + 0\mathbf{v} + 0\mathbf{w} = \mathbf{u} = \begin{bmatrix} -4 \\ 0 \\ -3 \end{bmatrix}, 0\mathbf{u} + 1\mathbf{v} + 0\mathbf{w} = \mathbf{v} = \begin{bmatrix} -2 \\ -1 \\ 5 \end{bmatrix},$

$0\mathbf{u} + 0\mathbf{v} + 1\mathbf{w} = \mathbf{w} = \begin{bmatrix} 9 \\ 6 \\ 11 \end{bmatrix}$

23. $a = 2$ and $b = 7$

25. $a = 3$, $b = 5$, and $c = 7$

27. $\mathbf{b} = 3\mathbf{a}_1 + 2\mathbf{a}_2$

29. $\mathbf{b} = 3\mathbf{a}_1 + 4\mathbf{a}_2$

31. 76 pounds of nitrogen, 31 pounds of phosphoric acid, and 14 pounds of potash.

33. Two bags of Vigoro and three bags of Parker's.

35. Three bags of Vigoro and two bags of Parker's.

37. No solution possible.

39. No solution possible.

41. Two cans of Red Bull and one can of Jolt Cola.

43. Two cans of Red Bull and two cans of Jolt Cola.

45. Three servings of Lucky Charms and five servings of Raisin Bran.

47. Two servings of Lucky Charms and three servings of Raisin Bran.

49. **(a)** $\mathbf{a} = \begin{bmatrix} 2000 \\ 8000 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3000 \\ 10{,}000 \end{bmatrix}$

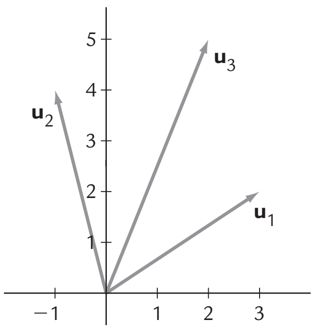
(b) $8\mathbf{b} = (8)\begin{bmatrix} 3000 \\ 10{,}000 \end{bmatrix} = \begin{bmatrix} 24{,}000 \\ 80{,}000 \end{bmatrix}.$

The company produces 24,000 computer monitors and 80,000 flat panel televisions at facility B in 8 weeks.

(c) 30,000 computer monitors and 108,000 flat panel televisions.

(d) 9 weeks of production at facility A and 2 weeks of production at facility B.

51. $\bar{\mathbf{v}} = \begin{bmatrix} \frac{8}{5} \\ \frac{16}{5} \end{bmatrix};$



53. 6kg of $\mathbf{u}_1$, 3kg of $\mathbf{u}_2$, 2kg of $\mathbf{u}_3$

55. For example, $\mathbf{u} = (0, 0, -1)$ and $\mathbf{v} = (3, 2, 0)$.

57. For example, $\mathbf{u} = (1, 0, 0)$, $\mathbf{v} = (1, 0, 0)$, and $\mathbf{w} = (-2, 0, 0)$.

59. For example, $\mathbf{u} = (1, 0)$ and $\mathbf{v} = (2, 0)$.

61. $x_1 = 3$ and $x_2 = -2$

63. True

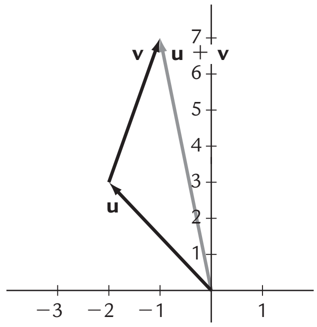
65. True

67. False

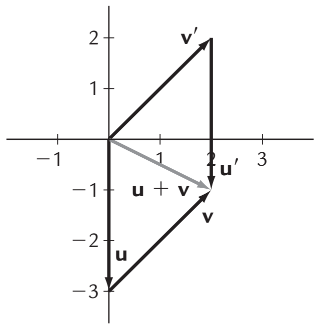
69. True

71. False

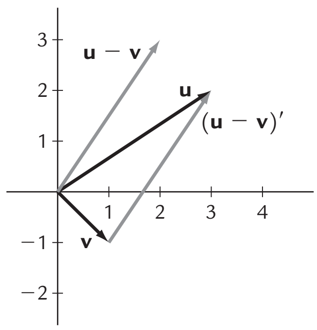
75.



77.



79.



81. $x_1 = 4$, $x_2 = -6.5$, and $x_3 = 1$.

Section 2.2

1. $0\mathbf{u}_1 + 0\mathbf{u}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, 1\mathbf{u}_1 + 0\mathbf{u}_2 = \begin{bmatrix} 2 \\ 6 \end{bmatrix}, 0\mathbf{u}_1 + 1\mathbf{u}_2 = \begin{bmatrix} 9 \\ 15 \end{bmatrix}$

3. $0\mathbf{u}_1 + 0\mathbf{u}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, 1\mathbf{u}_1 + 0\mathbf{u}_2 = \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix}, 0\mathbf{u}_1 + 1\mathbf{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$

5. $0\mathbf{u}_1 + 0\mathbf{u}_2 + 0\mathbf{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, 1\mathbf{u}_1 + 0\mathbf{u}_2 + 0\mathbf{u}_3 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix},$ $0\mathbf{u}_1 + 1\mathbf{u}_2 + 0\mathbf{u}_3 = \begin{bmatrix} 4 \\ 1 \\ 6 \end{bmatrix}$

7. $\mathbf{b}$ is not in the span of $\mathbf{a}_1$.

9. $\mathbf{b}$ is not in the span of $\mathbf{a}_1$.

11. $\mathbf{b}$ is not in the span of $\mathbf{a}_1$ and $\mathbf{a}_2$.

13. $A = \begin{bmatrix} 2 & 8 & -4 \\ -1 & -3 & 5 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -10 \\ 4 \end{bmatrix}$

15. $A = \begin{bmatrix} 1 & -1 & -3 & -1 \\ -2 & 2 & 6 & 2 \\ -3 & -3 & 10 & 0 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix}$

17. $x_1 \begin{bmatrix} 5 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 7 \\ -5 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \end{bmatrix}$

19. $x_1 \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ -5 \\ 8 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 7 \\ 2 \end{bmatrix} + x_4 \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 12 \\ 6 \\ 2 \end{bmatrix}$

21. Columns do not span $\mathbf{R}^2$.

23. Columns span $\mathbf{R}^2$.

25. Columns span $\mathbf{R}^3$.

27. Columns do not span $\mathbf{R}^3$.

29. For every choice of $\mathbf{b}$ there is a solution of $A\mathbf{x} = \mathbf{b}$.

31. There is a choice of $\mathbf{b}$ for which there is no solution to $A\mathbf{x} = \mathbf{b}$.

33. There is a choice of $\mathbf{b}$ for which there is no solution to $A\mathbf{x} = \mathbf{b}$.

35. Example: $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

37. Example: $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

39. $h \neq 3$

41. $h \neq 4$

43. Example: $\mathbf{u}_1 = (1, 0, 0)$, $\mathbf{u}_2 = (0, 1, 0)$, $\mathbf{u}_3 = (0, 0, 1)$, $\mathbf{u}_4 = (1, 1, 1)$

45. Example: $\mathbf{u}_1 = (1, 0, 0)$, $\mathbf{u}_2 = (2, 0, 0)$, $\mathbf{u}_3 = (3, 0, 0)$, $\mathbf{u}_4 = (4, 0, 0)$

47. Example: $\mathbf{u}_1 = (1, 0, 0)$, $\mathbf{u}_2 = (0, 1, 0)$

49. Example: $\mathbf{u}_1 = (1, -1, 0)$, $\mathbf{u}_2 = (1, 0, -1)$

51. True

53. False

55. False

57. True

59. False

61. True

63. False

65. (c) and (d) can possibly span $\mathbf{R}^3$.

67. HINT: Show that span $\{\mathbf{u}\} \subseteq$ span $\{c\mathbf{u}\}$ and that span $\{c\mathbf{u}\} \subseteq$ span $\{\mathbf{u}\}$

69. HINT: Let $S_1 = \{\mathbf{u}_1, \ldots, \mathbf{u}_k\}$ be a subset of S_2, and show that every linear combination $c_1\mathbf{u}_1 + \cdots c_k\mathbf{u}_k$ is in span(S_1).

71. HINT: Start with a linear combination $\mathbf{b} = c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3\mathbf{u}_3$. Show how to reorganize to write $\mathbf{b}$ as a linear combination of the set $\{\mathbf{u}_1 + \mathbf{u}_2, \mathbf{u}_1 + \mathbf{u}_3, \mathbf{u}_2 + \mathbf{u}_3\}$.

73. HINT: Generalize the argument given in Example 5.

75. True

77. False

Section 2.3

1. Linearly independent.

3. Linearly independent.

5. Linearly independent.

7. Linearly dependent.

9. Linearly independent.

11. Linearly independent.

13. System has only trivial solution.

15. System has only trivial solution.

17. System has only trivial solution.

19. Linearly dependent.

21. Linearly dependent.

23. Linearly dependent.

25. Vectors are linearly independent; none in span of the others.

27. Vectors are linearly independent; none in span of the others.

29. System does not have a unique solution for all $\mathbf{b}$.

31. System does not have a unique solution for all $\mathbf{b}$.

33. $\mathbf{u} = (1, 0, 0, 0)$, $\mathbf{v} = (0, 1, 0, 0)$, $\mathbf{w} = (1, 1, 0, 0)$

35. $\mathbf{u} = (1, 0)$, $\mathbf{v} = (2, 0)$, $\mathbf{w} = (3, 0)$

37. $\mathbf{u} = (1, 0, 0)$, $\mathbf{v} = (0, 1, 0)$, $\mathbf{w} = (1, 1, 0)$

39. False

41. False

43. False

45. False

47. True

49. False

51. False

53. (a), (b), and (c) can be linearly independent; (d) cannot.

55. HINT: Start by assuming that $\{c_1\mathbf{u}_1, c_2\mathbf{u}_2, c_3\mathbf{u}_3\}$ is linearly dependent, so the equation $x_1(c_1\mathbf{u}_1)+x_2(c_2\mathbf{u}_2)+x_3(c_3\mathbf{u}_3) = \mathbf{0}$ has a nontrivial solution. Show this implies that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is also linearly dependent, a contradiction.

57. HINT: Start by assuming that $\{\mathbf{u}_1+\mathbf{u}_2, \mathbf{u}_1+\mathbf{u}_3, \mathbf{u}_2+\mathbf{u}_3\}$ is linearly dependent, so the equation $x_1(\mathbf{u}_1+\mathbf{u}_2)+x_2(\mathbf{u}_1+\mathbf{u}_3)+x_3(\mathbf{u}_2+\mathbf{u}_3) = \mathbf{0}$ has a nontrivial solution. Show this implies that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is also linearly dependent, a contradiction.

59. HINT: Write the initial set of vectors as a nontrivial linear combination equal to $\mathbf{0}$ and then show that this linear combination can be extended to the new larger set of vectors.

61. HINT: If $\mathbf{u} = c\mathbf{v}$, then $\mathbf{u} - c\mathbf{v} = \mathbf{0}$.

63. HINT: Modify the proof of part (a) of Theorem 2.16.

65. No redundancy.

67. Linearly independent.

69. Linearly independent.

71. Unique solution for all $\mathbf{b}$.

73. Does not have a unique solution for all $\mathbf{b}$.

Chapter 3

Section 3.1

1. $T(\mathbf{u}_1) = \begin{bmatrix} -10 \\ 2 \end{bmatrix}, T(\mathbf{u}_2) = \begin{bmatrix} -4 \\ -33 \end{bmatrix}$

3. $T(\mathbf{u}_1) = \begin{bmatrix} -6 \\ 9 \end{bmatrix}, T(\mathbf{u}_2) = \begin{bmatrix} 16 \\ 11 \end{bmatrix}$

5. $\mathbf{y}$ is in the range of T.

7. $\mathbf{y}$ is in the range of T.

9. $T(-2\mathbf{u}_1 + 3\mathbf{u}_2) = \begin{bmatrix} -13 \\ 4 \end{bmatrix}$

11. $T(-\mathbf{u}_1 + 4\mathbf{u}_2 - 3\mathbf{u}_3) = \begin{bmatrix} 11 \\ -19 \end{bmatrix}$

13. Linear transformation, with $A = \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}$.

15. Not a linear transformation.

17. Linear transformation, with $A = \begin{bmatrix} -4 & 0 & 1 \\ 6 & 5 & 0 \end{bmatrix}$.

19. Linear transformation, with $A = \begin{bmatrix} 0 & \sin\frac{\pi}{4} \\ \ln 2 & 0 \end{bmatrix}$.

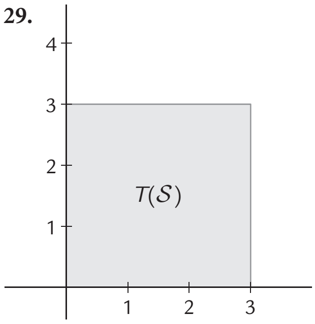
21. T is both one-to-one and onto.

23. T is not one-to-one but is onto.

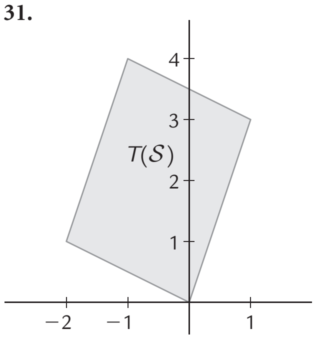
25. T is one-to-one but not onto.

27. T is neither one-to-one nor onto.

29.



31.



33. $T(\mathbf{x}) = \begin{bmatrix} 2 & 0 \\ 3 & 0 \end{bmatrix}\mathbf{x}$

35. $T(\mathbf{x}) = \begin{bmatrix} 7/3 & 0 \\ 0 & 0 \end{bmatrix}\mathbf{x}$

37. $T(\mathbf{x}) = \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}\mathbf{x}$

39. False

41. True

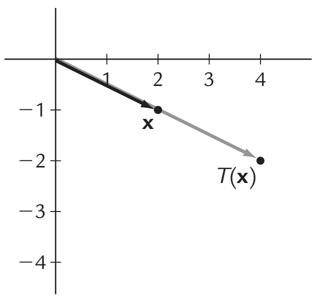
43. True

45. False

47. False

49. **(a)** $A = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$

(b)



51. HINT: Show that $T(\mathbf{x}+\mathbf{y}) = T(\mathbf{x})+T(\mathbf{y})$ and $T(r\mathbf{x}) = rT(\mathbf{x})$.

53. HINT: Let $T(\mathbf{x}) = A\mathbf{x}$, where A is a 2×3 matrix. Explain why $A\mathbf{x} = \mathbf{0}$ must have a nontrivial solution.

55. HINT: Show that $T(\mathbf{0}) = T(\mathbf{0}+\mathbf{0}) = T(\mathbf{0}) + T(\mathbf{0})$.

57. HINT: Use properties of matrix algebra.

59. HINT: Use the fact that T is one-to-one if and only if $T(\mathbf{x}) = \mathbf{0}$ has only the trivial solution.

61. HINT: Start by assuming that $x_1\mathbf{u}_1 + x_2\mathbf{u}_2 = \mathbf{0}$ has a nontrivial solution, and arrive at a contradiction.

63. HINT: Use hint given with problem.

65. HINT: The unit square consists of all vectors $\mathbf{x} = s\mathbf{u} + t\mathbf{v}$, where $\mathbf{u} = (1, 0)$, $\mathbf{v} = (0, 1)$, $0 \le s \le 1$, and $0 \le t \le 1$.

67. **(c)** $T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\mathbf{x}$

(d) T is neither one-to-one nor onto.

69. **(a)** $T(x^2 + \sin(x)) = 2x + \cos(x)$

71. $T\left(\begin{bmatrix} 5 \\ 3 \\ 6 \end{bmatrix}\right) = \begin{bmatrix} 603 \\ 565 \\ 766 \end{bmatrix}$

73. $T\left(\begin{bmatrix} 14 \\ 10 \\ 9 \end{bmatrix}\right) = \begin{bmatrix} 1255 \\ 1175 \\ 1609 \end{bmatrix}$

75. T is onto but not one-to-one.

77. T is neither one-to-one nor onto.

79. T is one-to-one but not onto.

Section 3.2

1. **(a)** $A + B = \begin{bmatrix} -3 & 5 \\ 0 & 4 \end{bmatrix}$

(b) $AB + I_2 = \begin{bmatrix} -1 & -7 \\ 2 & 4 \end{bmatrix}$

(c) $A + C$ is not possible.

3. **(a)** $(AB)^T = \begin{bmatrix} -2 & 2 \\ -7 & 3 \end{bmatrix}$

(b) CE is not defined.

(c) $(A - B)D = \begin{bmatrix} 3 & -15 & 12 \\ 16 & -30 & -6 \end{bmatrix}$

5. **(a)** $(C + E)B$ is not possible.

(b) $B\left(C^T + D\right) = \begin{bmatrix} -8 & 36 & 8 \\ -22 & 47 & 10 \end{bmatrix}$

(c) $E + CD = \begin{bmatrix} 6 & 4 & -20 \\ -11 & 21 & -4 \\ -3 & 17 & -6 \end{bmatrix}$

7. $a = -1$, $b = 1$, $c = -13$

9. $a = -1$, $b = 3$, $c = -3$, $d = 8$

11. $a = 2$

13. **(a)** $A = \begin{bmatrix} -6 & 52 \\ 4 & 17 \end{bmatrix}$

(b) $A = \begin{bmatrix} -24 & 53 \\ -10 & 35 \end{bmatrix}$

(c) $A = \begin{bmatrix} -1 & 50 \\ -20 & 39 \end{bmatrix}$

(d) $A = \begin{bmatrix} 4 & 27 \\ 0 & 25 \end{bmatrix}$

15. $A^2 - I$

17. $ABA - A^2 + B^3A - B^2A$

19. The right side assumes that $AB = BA$, which is not true in general.

21. The right side assumes that $AB = BA$, which is not true in general.

23. AB is 4×5.

25. **(a)** $A - B = \left[\begin{array}{cc|cc} -1 & -2 & 0 & 2 \\ 1 & -1 & -1 & 3 \\ \hline -1 & 3 & 0 & -3 \\ -2 & -1 & 3 & 3 \end{array}\right]$

(b) $AB = \left[\begin{array}{cc|cc} 14 & 5 & -6 & -10 \\ 4 & 7 & -4 & -7 \\ \hline -8 & 4 & 9 & -5 \\ -1 & 1 & -3 & 5 \end{array}\right]$

(c) $BA = \left[\begin{array}{cc|cc} 3 & -5 & 2 & 7 \\ -7 & 11 & 2 & -4 \\ \hline 4 & -1 & 9 & -1 \\ -1 & -8 & -2 & 12 \end{array}\right]$

27. **(a)** $B - A = \left[\begin{array}{ccc|c} 1 & 2 & 0 & -2 \\ \hline -1 & 1 & 1 & -3 \\ 1 & -3 & 0 & 3 \\ 2 & 1 & -3 & -3 \end{array}\right]$

(b) $AB = \left[\begin{array}{ccc|c} 14 & 5 & -6 & -10 \\ 4 & 7 & -4 & -7 \\ -8 & 4 & 9 & -5 \\ -1 & 1 & -3 & 5 \end{array}\right]$

(c) $BA + A = \left[\begin{array}{ccc|c} 4 & -7 & 1 & 10 \\ \hline -9 & 11 & 3 & 0 \\ 3 & 1 & 7 & -1 \\ -1 & -7 & 0 & 13 \end{array}\right]$

29. **(a)** $E = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(c) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

31. For example, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

33. For example, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

35. For example, $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$.

37. For example, $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

39. False

41. True

43. False

45. True

47. True

53. HINT: Start with $(AB)^T$, and use $A^T = A$, $B^T = B$ because A, B are symmetric.

55. **(a)** $A^T A$ is $m \times m$.

(b) HINT: Show $(A^T A)^T = A^T A$.

57. HINT: Follow hint given in exercise.

59. HINT: A proof by induction works well for this one.

61. **(a)** For example, $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$.

(b) HINT: Look at your example for part (a).

65. After one year: $\begin{bmatrix} 6500 \\ 2200 \\ 1300 \end{bmatrix}$; after two years: $\begin{bmatrix} 5375 \\ 2760 \\ 1865 \end{bmatrix}$;

after three years: $\approx \begin{bmatrix} 4531 \\ 3208 \\ 2261 \end{bmatrix}$;

after four years: $\approx \begin{bmatrix} 3898 \\ 3566 \\ 2535 \end{bmatrix}$.

67. Tomorrow: $\begin{bmatrix} 742 \\ 258 \end{bmatrix}$; the next day: $\approx \begin{bmatrix} 734 \\ 266 \end{bmatrix}$;

the day after that: $\approx \begin{bmatrix} 730 \\ 270 \end{bmatrix}$.

69. **(a)** $A + B = \begin{bmatrix} -4 & 1 & -3 & 5 \\ -5 & 5 & 3 & 2 \\ 6 & 11 & 0 & 5 \\ 13 & 2 & -1 & -2 \end{bmatrix}$

(b) $BA - I_4 = \begin{bmatrix} -26 & -15 & 4 & -31 \\ 5 & 1 & 9 & -28 \\ 14 & -11 & 11 & -12 \\ 4 & -9 & 13 & 24 \end{bmatrix}$

(c) $D + C$ is not possible.

71. **(a)** $AB = \begin{bmatrix} 25 & 22 & -14 & -1 \\ -23 & 10 & -1 & 21 \\ -68 & 36 & -21 & 34 \\ -31 & -3 & -12 & 0 \end{bmatrix}$

(b) $CD = \begin{bmatrix} 14 & 21 & 17 & 7 \\ 42 & 65 & 60 & 22 \\ 42 & 82 & 62 & 30 \\ 52 & 76 & 47 & 29 \end{bmatrix}$

(c) $(A - B)C^T$ is not possible.

73. **(a)** $(C + A)B$ is not possible.

(b) $C\left(C^T + D\right) = \begin{bmatrix} 21 & 40 & 41 & 29 \\ 61 & 120 & 124 & 84 \\ 66 & 146 & 182 & 106 \\ 74 & 138 & 123 & 109 \end{bmatrix}$

(c) $A + CD = \begin{bmatrix} 16 & 20 & 17 & 11 \\ 42 & 68 & 63 & 21 \\ 48 & 90 & 63 & 31 \\ 57 & 73 & 48 & 27 \end{bmatrix}$

Section 3.3

1. $\begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$

3. Inverse does not exist.

5. $\begin{bmatrix} 9 & -4 \\ -2 & 1 \end{bmatrix}$

7. Inverse does not exist.

9. $\begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$

11. Inverse does not exist.

13. $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

15. $\begin{bmatrix} 1 & -3 & -7 & 17 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

17. $x_1 = 35$ and $x_2 = -11$

19. $x_1 = -\frac{15}{4}$, $x_2 = \frac{29}{4}$ and $x_3 = \frac{5}{2}$

21. $T^{-1}(\mathbf{x}) = \begin{bmatrix} -2x_1 + 3x_2 \\ 3x_1 - 4x_2 \end{bmatrix}$

23. T^{-1} does not exist.

25. T^{-1} does not exist.

27. **(a)** $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

29. $\left[\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & 4 & -7 \\ 0 & -1 & 2 \end{array}\right]$

31. $\left[\begin{array}{rr|rr} 8 & -5 & 0 & 0 \\ -3 & 2 & 0 & 0 \\ \hline 0 & 0 & -3 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right]$

33. $\left[\begin{array}{rr|rrr} -8 & 3 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 & 0 \\ \hline -32 & 13 & 1 & -2 & 2 \\ 23 & -9 & 0 & 1 & 0 \\ 14 & -5 & 0 & 0 & 1 \end{array}\right]$

35. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

37. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

39. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

41. False

43. True

45. True

47. True

49. True

51. $X = A^{-1}B$

53. $X = C^{-1}B - A$

55. $A = \begin{bmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{bmatrix}$ and $A = \begin{bmatrix} a & b \\ \frac{1-a^2}{b} & -a \end{bmatrix}$ where $b \neq 0$.

57. HINT: Apply The Big Theorem.

59. $c \neq 0$ and $c \neq 1$

61. HINT: If A is $n \times n$ and not invertible, then the system $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution $\mathbf{x}_0$.

63. $B = C^{-1}AC$

65. HINT: Right-multiply by A^{-1}.

67. HINT: Since B is singular, there is a nontrivial solution to $B\mathbf{x} = \mathbf{0}$.

71. 6 J8's, 10 J40's, 8 J80's.

73. This combination is not possible.

75. 3 Vigoro, 4 Parker's, 5 Bleyer's.

77. 10 Vigoro, 14 Parker's, 11 Bleyer's.

79. "laptop"

81. "final exam"

83. $\begin{bmatrix} \frac{8}{145} & -\frac{14}{145} & -\frac{23}{145} & \frac{4}{29} \\ \frac{67}{145} & \frac{64}{145} & \frac{43}{145} & -\frac{10}{29} \\ -\frac{27}{145} & \frac{11}{145} & -\frac{13}{145} & \frac{1}{29} \\ -\frac{3}{29} & -\frac{2}{29} & \frac{5}{29} & \frac{7}{29} \end{bmatrix}$

85. Inverse does not exist.

Section 3.4

1. $a = 2$, $b = -14$

3. $a = 4$, $b = 3$, $c = 2$

5. $\mathbf{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

7. $\mathbf{x} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

9. $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

11. $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}$

13. $L = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$

15. $L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} -2 & -1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

17. $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -3 & 1 & 0 \\ 1 & 3 & -2 & 1 \end{bmatrix}$

$U = \begin{bmatrix} -1 & 0 & -1 & 2 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

19. $L = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} -1 & 2 & 1 & 3 \\ 0 & 1 & -3 & -5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

21. $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & -1 & -1 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

23. $L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 2 & -1 & 1 & 0 & 0 \\ -1 & 1 & -\frac{18}{17} & 1 & 0 \\ 2 & 0 & \frac{1}{17} & 0 & 1 \end{bmatrix}$

$U = \begin{bmatrix} -2 & 1 & 3 \\ 0 & 1 & 11 \\ 0 & 0 & 17 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

25. $L = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $U = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

27. $L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$, $U = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & \frac{1}{2} \end{bmatrix}$.

29. $L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

$U = \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & 1 \end{bmatrix}$

31. $E = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

33. $E = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

35. $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

37. $B = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

39. $B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

41. $B = \begin{bmatrix} 0 & 1 & 0 \\ -3 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

43. $E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$E^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{6} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \leftrightarrow \left\{-\frac{1}{6}R_2 \Rightarrow R_2\right\}$

45. $E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$E^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \leftrightarrow \{R_3 \Leftrightarrow R_4\}$

47. $E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -5 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$E^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \leftrightarrow \{5R_1 + R_2 \Rightarrow R_2\}$

49. $A^{-1} = \begin{bmatrix} \frac{7}{6} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

51. $A^{-1} = \begin{bmatrix} 17 & 1 & -2 & -7 \\ 8 & 0 & -1 & -4 \\ -6 & -2 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

53. $A^{-1} = \begin{bmatrix} 3 & -\frac{5}{3} & \frac{1}{3} \\ 3 & -\frac{3}{2} & \frac{1}{4} \\ 4 & -2 & \frac{1}{2} \end{bmatrix}$

55. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = LU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

57. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = LU = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

59. $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = LU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

61. False **63.** False **65.** False **67.** False

69. HINT: Apply properties of matrix multiplication.

71. HINT: Take each case separately and apply properties of matrix multiplication.

73. This matrix does not have an LU factorization.

75. $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ \frac{3}{2} & \frac{2}{7} & -\frac{1}{5} & 1 \end{bmatrix}$

$U = \begin{bmatrix} 10 & 2 & 0 & -4 & 2 \\ 0 & 0 & -14 & 7 & 21 \\ 0 & 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 & -16 \end{bmatrix}$

Section 3.5

1. Stochastic **3.** Stochastic

5. $a = 0.35$, $b = 0.55$

7. $a = \frac{8}{13}$, $b = \frac{1}{7}$, $c = \frac{1}{10}$

9. $a = 0.7$, $b = 0.7$

11. $a = 0.5$, $b = 0.4$, $c = 0.5$, $d = 0.4$

13. $\mathbf{x}_3 = \begin{bmatrix} 0.4432 \\ 0.5568 \end{bmatrix}$

15. $\mathbf{x}_3 = \begin{bmatrix} \frac{2531}{6750} \\ \frac{4219}{6750} \end{bmatrix}$

17. $\mathbf{x} = \begin{bmatrix} 0.71429 \\ 0.28571 \end{bmatrix} = \begin{bmatrix} 5/7 \\ 2/7 \end{bmatrix}$

19. $\mathbf{x} = \begin{bmatrix} 0.39807 \\ 0.29126 \\ 0.31067 \end{bmatrix}$

21. Not regular.

23. Not regular.

25. $A = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 & 0.3 \\ 0.4 & 0.4 & 0.4 & 0.4 \end{bmatrix}$

27. $A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

29. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $\mathbf{x}_0 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

31. False

33. False

35. False

37. Hint: Let $Y = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}$, show that $YA = Y$, and then show $Y(A\mathbf{x}) = 1$.

39. Hint: See exercise for hint.

43. Hint: Each column of A^{k+1} is a linear combination, with nonnegative scalars, of the columns of A^k.

45. Hints: **(b)** Compute A^2 then A^3, then look for a pattern. **(c)** $A^k \to \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ **(d)** $\mathbf{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

47. **(a)** $A = \begin{bmatrix} 0.9 & 0.15 \\ 0.1 & 0.85 \end{bmatrix}$

(b) Probability that the sixth person in the chain hears the wrong news is 0.32881.

(c) $\mathbf{x} = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$

49. **(a)** $A = \begin{bmatrix} 0.35 & 0.8 \\ 0.65 & 0.2 \end{bmatrix}$

(b) **i.** Probability that she will go to McDonald's two Sundays from now is 0.3575.

ii. Probability that she will go to McDonald's three Sundays from now is 0.48913.

(c) Probability that his third fast-food experience will be at Krusty's will be 0.521.

(d) $\mathbf{x} = \begin{bmatrix} 0.55173 \\ 0.44827 \end{bmatrix}$

51. **(a)** Probability that a book is at C after two more circulations is 0.21.

(b) Probability that the book is at B after three more circulations is 0.64.

(c) $\mathbf{x} = \begin{bmatrix} 0.17105 \\ 0.63158 \\ 0.19737 \end{bmatrix}$

53. $\mathbf{x}_9 = \mathbf{x}_{10} = \begin{bmatrix} 0.266666 \\ 0.399999 \\ 0.133333 \\ 0.200000 \end{bmatrix}$; the steady-state vector is

$\mathbf{x} = \begin{bmatrix} \frac{4}{15} \\ \frac{2}{5} \\ \frac{2}{15} \\ \frac{1}{5} \end{bmatrix}$

55. $A \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ so $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ has itself as its steady-state vector.

Also $A \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ so $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ has itself as its steady-state vector.

Chapter 4

Section 4.1

1. This is a subspace, equal to span $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

3. Not a subspace, because **0** is not in this set.

5. Not a subspace, because **0** is not in this set.

7. Not a subspace, because it is not closed under scalar multiplication.

9. Not a subspace, because it is not closed under addition.

11. Not a subspace, because it is not closed under scalar multiplication.

13. Not a subspace, because it is not closed under scalar multiplication.

15. A subspace, equal to null $\left(\begin{bmatrix} 1 & 1 & \cdots 1 \end{bmatrix}\right)$.

17. Not closed under scalar multiplication.

19. Not closed under addition.

21. $\text{null}(A) = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

23. $\text{null}(A) = \text{span}\left\{ \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix} \right\}$

25. $\text{null}(A) = \text{span}\left\{ \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix} \right\}$

27. $\text{null}(A) = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

29. $\text{null}(A) = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

31. $\text{null}(A) = \text{span}\left\{ \begin{bmatrix} 3 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

33. $\mathbf{b}$ is not in ker(T); $\mathbf{c}$ is in range(T).

35. $\mathbf{b}$ is not in ker(T); $\mathbf{c}$ is not in range(T).

37. For example, $S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x > 0 \right\}$.

39. For example, $S_1 = \left\{ \begin{bmatrix} x \\ 0 \end{bmatrix} : x \geq 0 \right\}$ and $S_2 = \left\{ \begin{bmatrix} x \\ 0 \end{bmatrix} : x < 0 \right\}$.

41. Let $T(\mathbf{x}) = A\mathbf{x}$, where $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$.

43. Let $T(\mathbf{x}) = A\mathbf{x}$, where $A = I_3$.

45. True

47. True

49. False

51. True

53. True

55. True

57. False

59. False

61. HINT: If $x \neq 0$ is in a subspace S, show that every real number must be in S.

63. HINT: Determine if $\mathbf{0}$ is in the set of solutions.

65. The vector $\mathbf{0}$ alone, lines and planes through the origin, and all of $\mathbf{R}^3$.

67. HINT: Determine if $\mathbf{0}$ is in the set of solutions.

69. HINT: Show that $\mathbf{x} \neq \mathbf{0}$ and $A\mathbf{x} = \mathbf{0}$ if and only if the columns of A are linearly dependent.

71. HINT: Note that $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-1)\mathbf{v}$.

73. span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$

75. span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix} \right\}$

77. span $\left\{ \begin{bmatrix} \frac{3}{7} \\ -\frac{13}{7} \\ \frac{5}{7} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{43}{56} \\ -\frac{5}{56} \\ -\frac{39}{56} \\ 0 \\ 1 \end{bmatrix} \right\}$

79. $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

Section 4.2

1. Not a basis, since $\mathbf{u}_1$ and $\mathbf{u}_2$ are not linearly independent.

3. Not a basis, since three vectors in a two-dimensional space must be linearly dependent.

5. Basis is $\left\{ \begin{bmatrix} 1 \\ -4 \end{bmatrix} \right\}$; dimension $= 1$.

7. Basis is $\left\{ \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 5 \end{bmatrix} \right\}$; dimension $= 2$.

9. Basis is $\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -5 \\ 1 \end{bmatrix} \right\}$; dimension $= 2$.

11. Basis is $\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -12 \end{bmatrix} \right\}$; dimension $= 2$.

13. Basis is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} \right\}$; dimension $= 3$.

15. Basis is $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \\ 9 \\ 7 \end{bmatrix} \right\}$; dimension $= 2$.

17. Basis is $\left\{ \begin{bmatrix} 2 \\ -6 \end{bmatrix} \right\}$; dimension is 1.

19. Basis is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$; dimension is 1.

21. Basis is $\left\{ \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$; dimension is 3.

23. One extension is $\left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$.

25. One extension is $\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$.

27. One extension is $\left\{ \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$.

29. null$(A) = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$. This subspace has no basis, and nullity $(A) = 0$.

31. The null space has basis $\left\{ \begin{bmatrix} -7 \\ 0 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$, and nullity$(A) = 2$.

33. For example, $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$.

35. For example, the span of the first m vectors of the n standard basis vectors of $\mathbf{R}^n$.

37. For example, $S_1 = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ and $S_2 = \text{span}\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

39. For example, $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

41. False

43. False

45. False

47. False

49. False

51. False

53. True

55. **(a)** 1, 2, or 3.
(b) 1 or 2.

57. HINT: Use the Big Theorem.

59. HINT: Show separately that the set is linearly independent and spans S.

61. HINT: A basis for S_1 can be expanded to a basis for S_2.

63. HINT: The entries below each pivot are equal to zero.

65. n

69. Subspace has basis $\left\{ \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ -2 \end{bmatrix} \right\}$, with dimension 2. The vectors are not a basis for $\mathbf{R}^3$.

71. Subspace has basis $\left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 7 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ -5 \\ 4 \end{bmatrix} \right\}$, with dimension 4. The vectors form a basis for $\mathbf{R}^4$.

73. Subspace has basis $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \\ 1 \\ 2 \end{bmatrix} \right\}$, with dimension 3. The vectors therefore do not span $\mathbf{R}^5$.

Section 4.3

1. Column space basis: $\left\{ \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 8 \end{bmatrix} \right\}$

Row space basis: $\left\{ \begin{bmatrix} 1 \\ 0 \\ -10 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} \right\}$

Null space basis: $\left\{ \begin{bmatrix} 10 \\ 4 \\ 1 \end{bmatrix} \right\}$

rank $= 2$, nullity $= 1$, $m = 3$

3. Column space basis: $\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

Row space basis: $\left\{ \begin{bmatrix} 1 \\ 0 \\ -4 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 5 \\ -1 \end{bmatrix} \right\}$

Null space basis: $\left\{ \begin{bmatrix} 4 \\ -5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

rank $= 2$, nullity $= 2$, $m = 4$

5. Column space basis: $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix} \right\}$

Row space basis: $\left\{ \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} \right\}$

Null space basis: $\left\{ \begin{bmatrix} -1 \\ \frac{1}{2} \\ 1 \end{bmatrix} \right\}$

rank $= 2$, nullity $= 1$, $m = 3$

7. Column space basis: $\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 11 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix} \right\}$

Row space basis: $\left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix} \right\}$

Null space basis: $\left\{ \begin{bmatrix} \frac{5}{3} \\ -\frac{1}{3} \\ -\frac{1}{3} \\ 1 \end{bmatrix} \right\}$

rank $= 3$, nullity $= 1$, $m = 4$

9. $x \neq 8$

11. $x = 18$

13. $\dim(\text{col}(A)) = 5$

15. $\dim(\text{row}(A)) = 3$, $\dim(\text{col}(A)) = 3$, $\text{nullity}(A) = 4$.

17. $\text{rank}(A) = 2$

19. $\text{nullity}(A) = 7$

21. $\dim(\text{range}(T)) = 4$

23. $\text{nullity}(A) = 0$

25. Maximum for rank$(A) = 5$, minimum for nullity$(A) = 8$.

27. rank$(A) = 3$

29. nullity$(A) = 2$

31. B has 3 nonzero rows.

33. A is 7×5.

35. For example, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

37. For example, $A = \left[\begin{array}{c|c} I_{3\times 3} & 0_{3\times 1} \\ \hline 0_{6\times 3} & 0_{6\times 1} \end{array}\right]$.

39. For example, $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

41. For example, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

43. True

45. True

47. False

49. False

51. False

53. True

55. HINT: row$(A) = $ col(A^T).

57. HINT: Apply the Rank–Nullity Theorem.

59. HINT: First suppose $n < m$ and apply the Rank–Nullity Theorem to A, then suppose that $m < n$ and apply the Rank–Nullity Theorem to A^T

61. rank$(A) = 2$, nullity$(A) = 3$.

63. rank$(A) = 2$, nullity$(A) = 1$.

Chapter 5

Section 5.1

1. $M_{23} = \begin{bmatrix} 7 & 0 \\ 5 & 1 \end{bmatrix}$, $M_{31} = \begin{bmatrix} 0 & -4 \\ 6 & 2 \end{bmatrix}$

3. $M_{23} = \begin{bmatrix} 6 & 1 & 5 \\ 7 & 1 & 1 \\ 4 & 3 & 2 \end{bmatrix}$, $M_{31} = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 3 & 0 \\ 3 & 1 & 2 \end{bmatrix}$

5. $M_{23} = \begin{bmatrix} 4 & 3 & 1 & 0 \\ 3 & 2 & 4 & 4 \\ 5 & 1 & 0 & 3 \\ 2 & 2 & 1 & 0 \end{bmatrix}$, $M_{31} = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 2 & 0 & 5 \\ 1 & 0 & 0 & 3 \\ 2 & 4 & 1 & 0 \end{bmatrix}$

7. $C_{13} = 4$, $C_{22} = -10$

9. $C_{13} = 1$, $C_{22} = 4$

11. $|A| = 60$; T is invertible.

13. $|A| = 20$; T is invertible.

15. $|A| = 51$; T is invertible.

17. $|A| = 8$; T is invertible.

19. $|A| = 14$

21. $|A|$ is not defined.

23. $|A| = -82$

25. The shortcut method does not apply.

27. $a = 9$

29. $a = 0$

31. $a = 4$

33. $a = 1$ or $a = 3$

35. $|A| = -8$ (A upper triangular)

37. $|A| = 0$ (column of zeros)

39. $|A| = 0$ (two equal rows)

41. $|A| = |A^T| = 11$

43. $|A| = |A^T| = 28$

45. $\lambda = -2$ or $\lambda = 7$

47. $\lambda = 1$

49. $\lambda = -2$, $\lambda = 1$, or $\lambda = 3$

51. $\lambda = 2$

53. (a) $|A| = 22$, determinant after row interchange $= -22$.
(b) $|A| = 1$, determinant after row interchange $= -1$.
Conjecture: Row interchanges change the sign of the determinant.

55. (a) $|A| = -13$, determinant after row interchange $= 13$.
(b) $|A| = 3$, determinant after row interchange $= -3$.
Conjecture: Row interchanges change the sign of the determinant.

57. (a) $|A| = 22$, determinant after multiplying row 1 by 3 is 66.
(b) $|A| = 1$, determinant after multiplying row 1 by 3 is 3.
Conjecture: Multiplying row 1 by 3 change the determinant by a factor of 3.

59. (a) $|A| = -13$, determinant after multiplying row 1 by 3 is -39.
(b) $|A| = 3$, determinant after multiplying row 1 by 3 is 9.
Conjecture: Multiplying row 1 by 3 changes the determinant by a factor of 3.

61. For example, $A = \begin{bmatrix} 12 & 0 \\ 0 & 1 \end{bmatrix}$.

63. For example, $A = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$.

65. For example, $A = \begin{bmatrix} 5 & -1 & \pi \\ e & 0 & 4 \\ 2 & 6 & -3 \end{bmatrix}$.

67. For example, $A = \begin{bmatrix} \pi & 0 & 5 \\ 8 & 1 & 0 \\ 0 & e & 1 \end{bmatrix}$.

69. False

71. False

73. False

75. False

77. HINT: Show that the determinant gives a linear equation in x and y, then plug in (x_1, y_1) and (x_2, y_2) separately to show they satisfy the equation.

79. HINT: Show that the given expression is equal to the determinant of the matrix obtained by replacing row j of A with row i.

81. HINT: Cofactor expansion along row or column of zeros.

83. $|A| = -26$

85. $|A| = 1215$

Section 5.2

1. $|A| = 2$

3. $|A| = 0$

5. $|A| = 1$

7. $|A| = 4$; A is invertible.

9. $|A| = 21$; A is invertible.

11. $|A| = 0$; A is not invertible.

13. $|A| = 8$; A is invertible.

15. Determinant $= -3$

17. Determinant $= -6$

19. $\det(AB) = \det(A)\det(B) = (-11)(3) = -33$
$\det(A + B) = -2 \neq -11 + 3 = \det(A) + \det(B)$

21. $\det(AB) = \det(A)\det(B) = (1)(-30) = -30$
$\det(A + B) = -76 \neq 1 - 30 = \det(A) + \det(B)$

23. (a) $|A^2| = 9$
(b) $|A^4| = 81$
(c) $|A^2A^T| = 27$
(d) $|A^{-1}| = \frac{1}{3}$

25. (a) $|A^2B^3| = -72$
(b) $|AB^{-1}| = -\frac{3}{2}$
(c) $|B^3A^T| = -24$
(d) $|A^2B^3B^T| = 144$

27. $|A| = 198$

29. $|A| = 4$

31. $|A| = 0$

33. $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = -3, |A||D| - |B||C| = -18$

35. Unique solution exists.

37. Unique solution exists.

39. Unique solution exists.

41. $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

43. $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} -1 & -2 \\ -2 & -4 \end{bmatrix}$

45. For example, $\det\left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}\right) = 1.$

47. False

49. False

51. False

53. True

55. True

57. HINT: Subtract one of the identical rows from the other, then apply cofactor expansion to the resulting matrix.

59. HINT: $|A| = |A^T|$.

61. HINT: Remove a factor of (-1) from each of the n rows.

63. HINT: $|A^2| = |A|^2$.

65. HINT: See hint given with problem.

67. HINT: Explain why a matrix can be transformed to echelon form without multiplying a row times a constant.

69. (a) HINT: E is diagonal, with a c for one diagonal entry and ones for the remaining diagonal entries.
(b) HINT: E is triangular, with ones along the diagonal.

71. HINT: See hint given with problem.

73. $|I_4 + AB| = |I_3 + BA| = -45,780$

Section 5.3

1. $x_1 = \frac{21}{8}, x_2 = \frac{3}{4}$

3. $x_1 = 9, x_2 = -17, x_3 = 1$

5. $x_1 = \frac{79}{49}, x_2 = \frac{22}{49}, x_3 = \frac{124}{49}$

7. $x_2 = \frac{11}{23}$

9. $x_2 = -\frac{25}{21}$

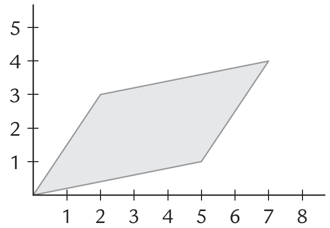
11. $x_2 = \frac{14}{39}$

13. $\text{adj}(A) = \begin{bmatrix} 7 & -5 \\ -3 & 2 \end{bmatrix}, A^{-1} = \begin{bmatrix} -7 & 5 \\ 3 & -2 \end{bmatrix}$

15. $\text{adj}(A) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

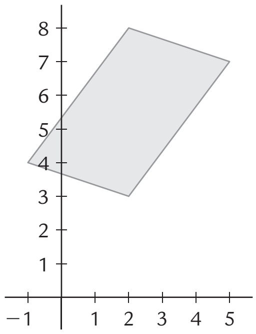
17. $\text{adj}(A) = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}, A^{-1} = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

19.



area $=13$

21.



area $=15$

23. area $(T(\mathcal{D})) = 165$

25. area $(T(\mathcal{D})) = 54$

27. area $(T(\mathcal{D})) = 54$

29. $T(\mathbf{x}) = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix} \mathbf{x}$ is one possible solution.

31. $T(\mathbf{x}) = \begin{bmatrix} \frac{3}{2}\sqrt{2} & 3\sqrt{2} \\ -\frac{3}{2}\sqrt{2} & 3\sqrt{2} \end{bmatrix} \mathbf{x}$ is one possible solution.

33. volume $= 80\pi$

35. volume $= 82$

37. For example,

$$\begin{aligned} x_1 + x_2 &= 1 \\ 2x_1 + 2x_2 &= 2 \end{aligned}$$

39. For example, let the parallelogram have vertices $(0, 0)$, $(5, 0)$, $(5, 1)$, and $(0, 1)$.

41. For example, $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$.

43. False

45. False

47. False

49. True

51. Hint: $|B| \neq 0$, so T is one-to-one. It remains to show that T is onto $\mathcal{R}$.

53. Hint: Use $|A|^{-1}\text{adj}(A) = A^{-1}$.

55. Hint: Show that the cofactor matrix of a symmetric matrix is also symmetric.

57. Hint: Consider the change in the cofactors when A is multiplied by c.

59. Hint: Start by replacing A with A^{-1} in $A = |A|^{-1}\text{adj}(A)$.

61. Hint: M_{ij} has a column (and row) of zeros when $i \neq j$.

63. $x_1 = \frac{1221}{752}$, $x_2 = \frac{811}{752}$, $x_3 = \frac{133}{94}$

65. $x_1 = \frac{704}{245}$, $x_2 = -\frac{14}{5}$, $x_3 = \frac{247}{245}$, $x_4 = -\frac{17}{49}$

67. $\text{adj}(A) = \begin{bmatrix} 27 & 53 & -15 \\ -72 & 41 & 40 \\ 59 & -26 & 28 \end{bmatrix}$,

$$A^{-1} = \begin{bmatrix} \frac{27}{547} & \frac{53}{547} & -\frac{15}{547} \\ -\frac{72}{547} & \frac{41}{547} & \frac{40}{547} \\ \frac{59}{547} & -\frac{26}{547} & \frac{28}{547} \end{bmatrix}$$

69. $\text{adj}(A) = \begin{bmatrix} -21 & -126 & 60 & -15 \\ -36 & 207 & -18 & 216 \\ 118 & 3 & -35 & -97 \\ 11 & -75 & 29 & -113 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} -\frac{7}{141} & -\frac{14}{47} & \frac{20}{141} & -\frac{5}{141} \\ -\frac{4}{47} & \frac{23}{47} & -\frac{2}{47} & \frac{24}{47} \\ \frac{118}{423} & \frac{1}{141} & -\frac{35}{423} & -\frac{97}{423} \\ \frac{11}{423} & -\frac{25}{141} & \frac{29}{423} & -\frac{113}{423} \end{bmatrix}$$

Chapter 6

Section 6.1

1. $\mathbf{x}_1$ is an eigenvector with associated eigenvalue $\lambda = -1$; $\mathbf{x}_2$ is not an eigenvector; $\mathbf{x}_3$ is an eigenvector with associated eigenvalue $\lambda = 4$.

3. $\mathbf{x}_1$ is an eigenvector with associated eigenvalue $\lambda = -1$; $\mathbf{x}_2$ is an eigenvector with associated eigenvalue $\lambda = 1$; $\mathbf{x}_3$ is an eigenvector with associated eigenvalue $\lambda = 2$.

5. $\mathbf{x}_1$ is an eigenvector with associated eigenvalue $\lambda = 3$; $\mathbf{x}_2$ is not an eigenvector; $\mathbf{x}_3$ is an eigenvector with associated eigenvalue $\lambda = 0$.

7. $\lambda = 3$ is not an eigenvalue of A.

9. $\lambda = -2$ is an eigenvalue of A.

11. A basis for the $\lambda = 4$ eigenspace is $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$.

13. A basis for the $\lambda = 2$ eigenspace is $\left\{ \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right\}$.

15. A basis for the $\lambda = 4$ eigenspace is $\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \right\}$.

17. A basis for the $\lambda = 6$ eigenspace is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

19. A basis for the $\lambda = -4$ eigenspace is $\left\{\begin{bmatrix} -3 \\ -5 \\ -2 \\ 3 \end{bmatrix}\right\}$.

21. $\det(A - \lambda I_2) = \lambda^2 + \lambda - 6$; basis for $\lambda = -3$ eigenspace is $\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$; basis for $\lambda = 2$ eigenspace is $\left\{\begin{bmatrix} 5 \\ 4 \end{bmatrix}\right\}$.

23. $\det(A - \lambda I_2) = \lambda^2 + 2\lambda + 1$;
basis for $\lambda = -1$ eigenspace is $\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$.

25. $\det(A - \lambda I_3) = -(\lambda - 2)(\lambda - 3)(\lambda + 1)$;
basis for $\lambda = 2$ eigenspace is $\left\{\begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix}\right\}$;
basis for $\lambda = 3$ eigenspace is $\left\{\begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}\right\}$;
basis for $\lambda = -1$ eigenspace is $\left\{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}$.

27. $\det(A - \lambda I_3) = -\lambda^3 + 3\lambda^2 - 2\lambda$
basis for $\lambda = 0$ eigenspace is $\left\{\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}\right\}$;
basis for $\lambda = 1$ eigenspace is $\left\{\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}\right\}$;
basis for $\lambda = 2$ eigenspace is $\left\{\begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}\right\}$.

29. $\det(A - \lambda I_4) = (\lambda + 2)(\lambda + 1)(\lambda - 1)^2$;
basis for $\lambda = -2$ eigenspace is $\left\{\begin{bmatrix} 0 \\ 3 \\ -3 \\ 1 \end{bmatrix}\right\}$;
basis for $\lambda = -1$ eigenspace is $\left\{\begin{bmatrix} 4 \\ 20 \\ -30 \\ 11 \end{bmatrix}\right\}$;
basis for $\lambda = 1$ eigenspace is $\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right\}$.

31. For example, $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

33. For example, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

35. For example, $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

37. False

39. False

41. True

43. True

45. False

47. **(a)** A is 6×6.
(b) $\lambda = 3, \lambda = 2$, and $\lambda = -1$.
(c) A is invertible.
(d) The largest possible dimension of an eigenspace is 3.

49. HINT: Apply the Big Theorem.

51. HINT: Explain why $\det(A - I_n) = 0$.

53. HINT: What is $A\mathbf{u}$ if $\mathbf{u}$ is associated with two distinct eigenvalues?

55. HINT: Which values of λ would *not* be eigenvalues?

57. HINT: Show $A^{-1}\mathbf{u} = \lambda^{-1}\mathbf{u}$.

59. HINT: Suppose that λ_1 is the eigenvalue of A associated with $\mathbf{u}$ and λ_2 is the eigenvalue of B associated with $\mathbf{u}$. Determine $AB\mathbf{u}$.

61. HINT: What is $A\mathbf{u}$ when $\mathbf{u} = (1, 1, \ldots, 1)$?

63. HINT: Note that $\det(A - \lambda I_n) = \det\left((A - \lambda I_n)^T\right)$.

65. HINT: What is $A\mathbf{u}$ when $\mathbf{u} = (1, 1, \ldots, 1)$?

67. Basis for $\lambda = 1$ eigenspace is $\left\{\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}\right\}$;
basis for $\lambda = 2$ eigenspace is $\left\{\begin{bmatrix} -1 \\ 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 9 \\ 0 \\ 2 \end{bmatrix}\right\}$.

69. Basis for $\lambda = 0$ eigenspace is $\left\{\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}\right\}$;
basis for $\lambda = 1$ eigenspace is $\left\{\begin{bmatrix} -1 \\ 0 \\ 3 \\ -1 \\ 1 \end{bmatrix}\right\}$;
basis for $\lambda = 2$ eigenspace is $\left\{\begin{bmatrix} -1 \\ -2 \\ 2 \\ -1 \\ 1 \end{bmatrix}\right\}$;
basis for $\lambda = -2$ eigenspace is $\left\{\begin{bmatrix} -1 \\ -3 \\ 3 \\ -2 \\ 1 \end{bmatrix}\right\}$;

basis for $\lambda = -1$ eigenspace is $\left\{\begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix}\right\}$.

Section 6.2

1. $\mathbf{x}_1 = \begin{bmatrix}1\\1\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}-2\\6\end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix}-20\\28\end{bmatrix}$

3. $\mathbf{x}_1 = \begin{bmatrix}6\\4\\4\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}52\\48\\48\end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix}504\\496\\496\end{bmatrix}$

5. $\mathbf{x}_1 = \begin{bmatrix}3\\0\\-3\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}9\\0\\-9\end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix}27\\0\\-27\end{bmatrix}$

7. $\mathbf{x}_1 = \begin{bmatrix}-1\\1\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}1.00\\-0.33\end{bmatrix}$

9. $\mathbf{x}_1 = \begin{bmatrix}0.00\\-0.50\\1.00\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}1.00\\-0.25\\1.00\end{bmatrix}$

11. $\mathbf{x}_1 = \begin{bmatrix}0.00\\1.00\\-0.67\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}0.00\\1.00\\-0.56\end{bmatrix}$

13. The Power Method will converge, with eigenvalue $\lambda = 7$.

15. The Power Method will converge, with eigenvalue $\lambda = -6$.

17. The Power Method will converge, with eigenvalue $\lambda = 6$.

19. $B = \begin{bmatrix}-3 & 2\\3 & -2\end{bmatrix}$

21. $B = \begin{bmatrix}-10 & 2 & -7\\-10 & -7 & 2\\-10 & 2 & -7\end{bmatrix}$

23. $B = \begin{bmatrix}-7 & 1\\5 & -2\end{bmatrix}$

25. $B = \begin{bmatrix}4 & 1 & 4\\1 & 6 & 9\\2 & 6 & 2\end{bmatrix}$

27. $\lambda = \frac{1}{1/4} = 4$; the eigenvector is $\begin{bmatrix}1\\1/2\\0\end{bmatrix}$.

29. For example, $A = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$ and $\mathbf{x}_0 = \begin{bmatrix}1\\0\end{bmatrix}$.

31. For example, $A = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ and $\mathbf{x}_0 = \begin{bmatrix}1\\0\end{bmatrix}$.

33. For example, $A = \begin{bmatrix}0 & -1\\1 & -1\end{bmatrix}$ and $\mathbf{x}_0 = \begin{bmatrix}1\\0\end{bmatrix}$.

35. False

37. True

39. True

41. $\mathbf{x}_1 = \begin{bmatrix}1\\-1\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}0\\1\end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix}1\\-1\end{bmatrix}, \mathbf{x}_4 = \begin{bmatrix}0\\1\end{bmatrix}$.
The sequence $\mathbf{x}_k$ does not converge, it alternates. The eigenvalues of A are $\lambda = 1$ and $\lambda = -1$, and so there is no dominant eigenvalue, and convergence is not assured.

43. $\mathbf{x}_1 = \begin{bmatrix}-\frac{1}{2}\\1\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}-\frac{1}{2}\\1\end{bmatrix}, \ldots$ and the sequence converges to the eigenvalue $\lambda = 1$ because $\mathbf{x}_0 = \begin{bmatrix}-1\\2\end{bmatrix}$ is an eigenvector associated with $\lambda = 1$.

45. $\mathbf{x}_1 = \begin{bmatrix}1\\1\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}-2\\6\end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix}-20\\28\end{bmatrix}, \mathbf{x}_4 = \begin{bmatrix}-104\\120\end{bmatrix},$
$\mathbf{x}_5 = \begin{bmatrix}-464\\496\end{bmatrix}, \mathbf{x}_6 = \begin{bmatrix}-1952\\2016\end{bmatrix}$

47. $\mathbf{x}_1 = \begin{bmatrix}6\\4\\4\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}52\\48\\48\end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix}504\\496\\496\end{bmatrix}, \mathbf{x}_4 = \begin{bmatrix}5008\\4992\\4992\end{bmatrix},$
$\mathbf{x}_5 = \begin{bmatrix}50{,}016\\49{,}984\\49{,}984\end{bmatrix}, \mathbf{x}_6 = \begin{bmatrix}500{,}032\\499{,}968\\499{,}968\end{bmatrix}$

49. $\mathbf{x}_1 = \begin{bmatrix}3\\0\\-3\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}9\\0\\-9\end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix}27\\0\\-27\end{bmatrix}, \mathbf{x}_4 = \begin{bmatrix}81\\0\\-81\end{bmatrix},$
$\mathbf{x}_5 = \begin{bmatrix}243\\0\\-243\end{bmatrix}, \mathbf{x}_6 = \begin{bmatrix}729\\0\\-729\end{bmatrix}$.

51. $\lambda = 2$; eigenvector $= \begin{bmatrix}-1\\0\end{bmatrix}$.

53. $\lambda = 4.2458$; eigenvector $= \begin{bmatrix}-0.0579\\1.0000\\-0.6518\end{bmatrix}$.

55. $\lambda = 3$; eigenvector $= \begin{bmatrix}0\\1\\-0.5\end{bmatrix}$.

Section 6.3

1. $\mathbf{x} = \begin{bmatrix}1\\-7\end{bmatrix}$

3. $\mathbf{x} = \begin{bmatrix}0\\2\end{bmatrix}$

5. $\mathbf{x} = \begin{bmatrix}1\\1\\2\end{bmatrix}$

7. $\mathbf{x}_{\mathcal{B}} = \begin{bmatrix}10\\-7\end{bmatrix}$

9. $\mathbf{x}_{\mathcal{B}} = \begin{bmatrix}2\\1\end{bmatrix}$

11. $\mathbf{x}_{\mathcal{B}} = \begin{bmatrix}4\\-1\\-4\end{bmatrix}$

13. $\begin{bmatrix} -5 & -1 \\ 9 & 2 \end{bmatrix}$

15. $\begin{bmatrix} -19 & -4 & 16 \\ 15 & 1 & -14 \\ 6 & 2 & -5 \end{bmatrix}$

17. $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

19. $\begin{bmatrix} -2 & -1 \\ 9 & 5 \end{bmatrix}$

21. $\begin{bmatrix} 31 & -15 & -49 \\ 17 & -7 & -25 \\ -6 & 3 & 10 \end{bmatrix}$

23. $\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$

25. $\mathbf{x}_{\mathcal{B}_2} = \begin{bmatrix} -9 \\ 16 \end{bmatrix}_{\mathcal{B}_2}$

27. $\mathbf{x}_{\mathcal{B}_2} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}_{\mathcal{B}_2}$

29. $\mathbf{x}_{\mathcal{B}_1} = \begin{bmatrix} -193 \\ -99 \\ 39 \end{bmatrix}_{\mathcal{B}_1}$

31. $\begin{bmatrix} a \\ b \end{bmatrix}_{\mathcal{B}_1} = \begin{bmatrix} b \\ a \end{bmatrix}_{\mathcal{B}_2}$

33. For example, $\mathcal{B} = \left\{ \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1/3 \end{bmatrix} \right\}$.

35. For example, $\mathcal{B}_1 = \left\{ \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1/2 \end{bmatrix} \right\}$

and $\mathcal{B}_2 = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$.

37. For example, $\mathcal{B}_1 = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \end{bmatrix} \right\}$

and $\mathcal{B}_2 = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$.

39. True

41. True

43. False

45. HINT: Write **u** and **v** in terms of the basis vectors of $\mathcal{B}$.

47. HINT: Focus on showing that the two properties required of a linear transformation both hold.

49. HINT: Explain why for each column $\mathbf{u}_i$ of U, the product $V^{-1}\mathbf{u}_i = [\mathbf{u}_i]_{\mathcal{B}_2}$.

51. $\begin{bmatrix} \frac{61}{35} & \frac{48}{5} & -\frac{316}{35} \\ \frac{17}{7} & 8 & -\frac{51}{7} \\ \frac{62}{35} & \frac{51}{5} & -\frac{382}{35} \end{bmatrix}$

53. $\begin{bmatrix} -\frac{49}{538} & \frac{55}{269} & \frac{3}{538} & \frac{364}{269} \\ \frac{37}{538} & \frac{211}{269} & \frac{393}{538} & \frac{340}{269} \\ -\frac{45}{269} & \frac{112}{269} & -\frac{129}{269} & -\frac{369}{269} \\ \frac{219}{538} & -\frac{416}{269} & \frac{305}{538} & -\frac{205}{269} \end{bmatrix}$

55. $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ -1 & -1 & -1 \end{bmatrix}$

Section 6.4

1. $A^5 = \begin{bmatrix} 131 & -396 \\ 33 & -100 \end{bmatrix}$

3. $A^5 = \begin{bmatrix} 1 & -93 & -184 \\ 0 & 32 & 66 \\ 0 & 0 & -1 \end{bmatrix}$

5. $A = \begin{bmatrix} 19 & -12 \\ 30 & -19 \end{bmatrix}$

7. $A = \begin{bmatrix} 2 & -3 & 4 \\ 0 & -1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$

9. The matrix is not diagonalizable.

11. $P = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$

13. $P = \begin{bmatrix} \frac{1}{3} & \frac{1}{2} & \frac{1}{2} \\ -\frac{2}{3} & -\frac{1}{2} & -1 \\ 1 & 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

15. $P = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

17. $P = \begin{bmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$

19. $A^{1000} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

21. $A^{1000} = \begin{bmatrix} 2\left(3^{1000}\right) - 1 & 2 - 2\left(3^{1000}\right) \\ 3^{1000} - 1 & 2 - 3^{1000} \end{bmatrix}$

23. Dimension $= 2$.

25. For example, $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

27. For example, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

29. For example, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ has eigenvalues 0, 1, and 2.

31. True

33. False

35. False

37. False

39. HINT: $\mathbf{u}_1$ and $\mathbf{u}_2$ must be linearly independent.

41. HINT: Each eigenvalue has infinitely many distinct associated eigenvectors.

43. HINT: What is A^T if $A = PDP^{-1}$?

45. HINT: Let $A = PD_1P^{-1}$ and $B = PD_2P^{-1}$, then show that $AB = BA$.

47. $P = \begin{bmatrix} -1 & -\frac{2}{3} & -\frac{1}{3} & 0 \\ 2 & \frac{2}{3} & \frac{1}{3} & 1 \\ -3 & -2 & -\frac{2}{3} & 0 \\ 2 & 2 & \frac{2}{3} & 0 \end{bmatrix}$

$D = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

49. $P = \begin{bmatrix} 0 & 0 & \frac{2}{3} & 2 & 4 \\ 4 & 0 & -\frac{2}{3} & -2 & -8 \\ 4 & 1 & \frac{4}{3} & 4 & 8 \\ 4 & 0 & -2 & -4 & -8 \\ 0 & 0 & \frac{4}{3} & 2 & 4 \end{bmatrix}$

$D = \begin{bmatrix} -2 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$

Section 6.5

1. **(a)** $8 + 2i$
(b) $19 + 8i$
(c) $2 - 6i$
(d) $23 + 14i$

3. $\lambda_1 = 2 + 2i$; eigenspace basis $= \left\{ \begin{bmatrix} 1 + 2i \\ -5 \end{bmatrix} \right\}$

$\lambda_2 = 2 - 2i$; eigenspace basis $= \left\{ \begin{bmatrix} 1 - 2i \\ -5 \end{bmatrix} \right\}$

5. $\lambda_1 = 2 + i$; eigenspace basis $= \left\{ \begin{bmatrix} 1 - i \\ -1 \end{bmatrix} \right\}$

$\lambda_2 = 2 - i$; eigenspace basis $= \left\{ \begin{bmatrix} 1 + i \\ -1 \end{bmatrix} \right\}$

7. $\lambda_1 = 3 + i$; eigenspace basis $= \left\{ \begin{bmatrix} 1 + i \\ -1 \end{bmatrix} \right\}$

$\lambda_2 = 3 - i$; eigenspace basis $= \left\{ \begin{bmatrix} 1 - i \\ -1 \end{bmatrix} \right\}$

9. Rotation is by $\tan^{-1}(1/2) \approx 0.4636$ radians; the dilation is by $\sqrt{5}$.

11. Rotation is by $\tan^{-1}(1) = \frac{\pi}{4}$ radians; the dilation is by $\sqrt{2}$.

13. Rotation is by $\tan^{-1}(-3/4) \approx -0.6435$ radians; the dilation is by 5.

15. The rotation–dilation matrix is $B = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$.

17. The rotation–dilation matrix is $B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$.

19. The rotation–dilation matrix is $B = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$.

21. Other roots are $1-2i$ and $3+i$, the multiplicity of each root is 1.

23. For example, $z = \frac{6\sqrt{5}}{5} + \frac{3\sqrt{5}}{5}i$.

25. $B = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

27. For example, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & -4 \\ 2 & -1 \end{bmatrix}$.

29. For example, $A = \begin{bmatrix} i & i \\ -i & -i \end{bmatrix}$.

31. True

33. False

35. False

37. True

39. True

41. HINT: Start with $z = x + iy$ and $w = u + iv$, then apply the properties of complex conjugation.

43. HINT: Apply Exercise 41(b).

45. HINT: Start with $\lambda = x + iy$, then apply the properties of complex conjugation.

47. HINT: $|A - \lambda I| = (a - \lambda)^2 + b^2$.

49. **(a)** HINT: Write $\mathbf{u} = \text{Re}(\mathbf{u}) + i\text{Im}(\mathbf{u})$.
(b) HINT: Use hint given with this part of problem.
(c) HINT: Show that the real and imaginary parts of AP and PC are the same.

51. $\lambda_{1,2} = -2.507 \pm 1.692i \Rightarrow \left\{ \begin{bmatrix} 0.2373 \pm 0.3607i \\ 0.0862 \mp 0.2878i \\ -0.8505 \end{bmatrix} \right\}$

$\lambda_3 = 6.013 \Rightarrow \left\{ \begin{bmatrix} 0.5837 \\ 0.6889 \\ 0.4298 \end{bmatrix} \right\}$

53. $\lambda_{1,2} = 2.5948 \pm 0.4119i \Rightarrow \left\{ \begin{bmatrix} 0.6638 \\ 0.1906 \pm 0.2156i \\ 0.0472 \mp 0.2804i \\ -0.6280 \mp 0.0368i \end{bmatrix} \right\}$

$\lambda_3 = 13.2693 \Rightarrow \left\{ \begin{bmatrix} 0.0639 \\ 0.1441 \\ 0.3504 \\ 0.9232 \end{bmatrix} \right\}$

$$\lambda_4 = -5.4589 \Rightarrow \left\{ \begin{bmatrix} 0.6812 \\ -0.3472 \\ -0.5476 \\ 0.3399 \end{bmatrix} \right\}$$

Section 6.6

1. $y_1 = c_1 e^{-t} + c_2 e^{2t}$
$y_2 = c_1 e^{-t} - c_2 e^{2t}$

3. $y_1 = 4c_1 e^{2t} + c_2 e^{-2t} + 2c_3 e^{-2t}$
$y_2 = 3c_1 e^{2t} + 2c_2 e^{-2t} + 3c_3 e^{-2t}$
$y_3 = c_1 e^{2t} + c_3 e^{-2t}$

5. $y_1 = c_1 (\cos 2t - \sin 2t) + c_2 (\cos 2t + \sin 2t)$
$y_2 = c_1 (2\cos 2t + \sin 2t) - c_2 (\cos 2t - 2\sin 2t)$

7. $y_1 = 3c_1 e^{4t} - c_2 (\sin t - 4\cos t)\, e^t + c_3 (\cos t + 4\sin t)\, e^t$
$y_2 = c_1 e^{4t} - 2c_3 (\cos t)\, e^t + 2c_2 (\sin t)\, e^t$
$y_3 = 5c_1 e^{4t} - c_2 (\sin t - 3\cos t)\, e^t + c_3 (\cos t + 3\sin t)\, e^t$

9. $y_1 = 6c_1 e^t + c_2 e^{4t} + c_3 (3\cos t - 2\sin t)\, e^t +$
$c_4 (2\cos t + 3\sin t)\, e^t$
$y_2 = 2c_1 e^t + 2c_2 e^{4t} + 6c_3 (\cos t)\, e^t + 6c_4 (\sin t)\, e^t$
$y_3 = 5c_1 e^t + 3c_2 e^{4t} + c_3 (2\cos t + 3\sin t)\, e^t -$
$c_4 (3\cos t - 2\sin t)\, e^t$
$y_4 = 2c_2 e^{4t} + 5c_3 (\sin t)\, e^t - 5c_4 (\cos t)\, e^t$

11. $y_1 = 2c_1 e^{-t} + 2c_2 e^{3t}$
$y_2 = -c_1 e^{-t} + c_2 e^{3t}$

13. $y_1 = c_1 e^{-t} + 2c_2 e^{3t}$
$y_2 = c_1 e^{-t} + c_2 e^{3t}$

15. $y_1 = -c_1 (\cos 2t - 2\sin 2t)\, e^{2t} - c_2 (2\cos 2t + \sin 2t)\, e^{2t}$
$y_2 = 5c_1 (\cos 2t)\, e^{2t} + 5c_2 (\sin 2t)\, e^{2t}$

17. $y_1 = 2c_1 + c_2 e^{-t} - c_3 e^t$
$y_2 = c_1 + 3c_3 e^t$
$y_3 = 2c_1 + c_2 e^{-t}$

19. $y_1 = -2e^{-2t} + 6e^{3t}$
$y_2 = -2e^{-2t} + 3e^{3t}$

21. $y_1 = -(\sin 3t)\, e^t + 2(\cos 3t)\, e^t$
$y_2 = -(\cos 3t)\, e^t - 2(\sin 3t)\, e^t$

23. $y_1 = 2e^t - e^{2t} - 2e^{-t}$
$y_2 = -2e^t + 2e^{2t}$
$y_3 = 4e^t - 2e^{2t} - 6e^{-t}$

25. $y_1 = c_1 e^{(-0.3)t} + 2c_2 e^{(-0.4)t}$
$y_2 = -c_1 e^{(-0.3)t} - 3c_2 e^{(-0.4)t}$

27. $y_1 = 70e^{-0.3t} - 60e^{-0.4t}$
$y_2 = 90e^{-0.4t} - 70e^{-0.3t}$

29. $y_1 = -c_1 e^{-8t} + 5c_2 e^t$, and $y_2 = c_1 e^{-8t} + 4c_2 e^t$. As t gets large, $y_1 \approx 5c_2 e^t$ and $y_2 \approx 4c_2 e^t$, and hence the ratio $y_1/y_2 \approx 5/4$.

31. $y_1 = \frac{5}{3}e^t - \frac{2}{3}e^{-8t}$, $y_2 = \frac{4}{3}e^t + \frac{2}{3}e^{-8t}$

33. For example, $y_1' = -3y_1$ and $y_2' = 2y_2$.

35. For example,

$$\begin{aligned} y_1' &= 10y_1 + 6y_2 \\ y_2' &= -18y_1 - 11y_2 \end{aligned}$$

37. For example,

$$\begin{aligned} y_1' &= -6y_1 + 4y_2 + 7y_3 \\ y_2' &= -7y_1 + 5y_2 + 7y_3 \\ y_3' &= -4y_1 + 4y_2 + 5y_3 \end{aligned}$$

39. True

41. False

43. $y_1 \approx -0.7811c_1 e^{7.065t} - 0.7041c_2 (\cos 2.580t)\, e^{(-3.033t)}$
$- 0.7041c_3 (\sin 2.580t)\, e^{(-3.033t)}$
$y_2 \approx 0.4471c_1 e^{7.065t} - c_2(0.1528 \cos(2.580t)$
$+0.4597 \sin(2.580t))e^{(-3.033t)} - c_3(0.1528 \sin(2.580t)$
$-0.4597 \cos(2.580t))e^{(-3.033t)}$
$y_3 \approx -0.4359c_1 e^{7.065t} + c_2(0.5141 \cos(2.580t)$
$+0.0729 \sin(2.580t))e^{(-3.033t)} + c_3(0.5141 \sin(2.580t)$
$-0.0729 \cos(2.580t))e^{(-3.033t)}$

45. $y_1 \approx -0.8167c_1 e^{-4.114t} + 1.139c_2 e^{7.297t}$
$+ c_3 (0.2576 \sin(4.698t) - 0.5848 (\cos 4.698t))\, e^{1.408t}$
$- c_4 (0.2576 \cos(4.698t) + 0.5848 \sin(4.698t))\, e^{1.408t}$
$y_2 \approx -0.8101c_1 e^{-4.114t} + 0.1064c_2 e^{7.297t}$
$- c_3 (2.336 \sin(4.698t) - 2.858 \cos(4.698t))\, e^{1.408t}$
$+ c_4 (2.336 \cos(4.698t) + 2.858 \sin(4.698t))\, e^{1.408t}$
$y_3 \approx 0.5896c_1 e^{-4.114t} + 0.39c_2 e^{7.297t}$
$- c_3 e^{1.408t} (2.385 \cos(4.698t) + 1.314 \sin(4.698t))$
$- c_4 e^{1.408t} (2.385 \sin(4.698t) - 1.314 \cos(4.698t))$
$y_4 \approx c_1 e^{-4.114t} + c_2 e^{7.297t}$
$+ c_3 \cos(4.698t)\, e^{1.408t} + c_4 \sin(4.698t)\, e^{1.408t}$

47. $y_1 \approx 0.5746e^{8.01t} - 0.3412e^{1.106t} - 1.233e^{-7.115t}$
$y_2 \approx -0.03121e^{8.01t} + 0.1231e^{1.106t} - 4.092e^{-7.115t}$
$y_3 \approx 0.7118e^{8.01t} + 0.1924e^{1.106t} + 2.096e^{-7.115t}$

49. $y_1 \approx 11.63 (\cos 2.153t)\, e^{-3.179t} - 0.4729e^{12.53t}$
$- 4.158e^{-0.1732t} + 7.978 (\sin 2.153t)\, e^{-3.179t}$
$y_2 \approx 8.266 (\cos 2.153t)\, e^{-3.179t} - 0.3908e^{12.53t}$
$- 5.876e^{-0.1732t} + 6.113 (\sin 2.153t)\, e^{-3.179t}$
$y_3 \approx 2.928e^{-0.1732t} - 0.1355e^{12.53t}$
$- 4.792 (\cos 2.153t)\, e^{-3.179t} + 3.693 (\sin 2.153t)\, e^{-3.179t}$
$y_4 \approx 4.349e^{-0.1732t} - 1.249e^{12.53t}$
$- 8.101 (\cos 2.153t)\, e^{-3.179t} - 7.601 (\sin 2.153t)\, e^{-3.179t}$

Chapter 7

Section 7.1

1. HINT: The required properties follow from the same properties of the real numbers.

3. HINT: You may assume that the sum of two continuous functions is a continuous function, as is the scalar multiple of a continuous function.

5. HINT: Adding two polynomials cannot produce a polynomial of degree greater than that of those being added. The scalar multiple of a polynomial produces a new polynomial that has the same degree or is equal to zero.

7. HINT: The hint from Exercise 3 applies here.

9. V is not a vector space under the given arithmetic operations. For instance, there is no vector $\mathbf{0}$ such that $\mathbf{v} + \mathbf{0} = \mathbf{v}$ for all $\mathbf{v}$.

11. V is not a vector space. Property 5(d) does not always hold. For instance, $(1+0)\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$ but $(1)\begin{bmatrix}1\\0\end{bmatrix} + (0)\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$.

13. HINT: Show that the three requirements for a subspace are met.

15. HINT: Show that the three requirements for a subspace are met.

17. HINT: Show that the three requirements for a subspace are met.

19. S is a subspace.

21. S is a subspace.

23. S is not a subspace. S is not closed under addition.

25. S is a subspace.

27. S is not a subspace. The zero vector is not in S.

29. S is a subspace.

31. S is not a subspace.

33. For example, the set of vectors in the first quadrant of $\mathbf{R}^2$, with the usual definition of addition and scalar multiplication.

35. For example, the set of vectors in the first quadrant of $\mathbf{R}^2$, with the usual definition of addition and scalar multiplication.

37. Aside from $V_1 = \mathbf{R}^n$ with the usual definition of addition and scalar multiplication, we can also have $V_2 = \mathbf{R}^n$, but we let $\mathbf{w}$ be a fixed vector and then define addition by $\mathbf{u} \oplus \mathbf{v} = \mathbf{u} + \mathbf{v} - \mathbf{w}$ and scalar multiplication by $c \odot \mathbf{u} = c(\mathbf{u} - \mathbf{w}) + \mathbf{w}$. In this case, $\mathbf{w}$ is the zero vector for V_2.

39. True

41. True

43. True

45. True

47. HINT: Construct $\mathbf{0}$ by using $\mathbf{u}$ in S (S is nonempty) and observing that there must be a corresponding $-\mathbf{u}$ in S.

49. HINTS:
(a) Use the fact that addition of vectors is commutative.
(b) Assume that there are two zero vectors $\mathbf{0}_a$ and $\mathbf{0}_b$, then show that $\mathbf{0}_a = \mathbf{0}_b$.
(c) Use $\mathbf{v} + 0 \cdot \mathbf{v} = (1+0)\mathbf{v} = \mathbf{v}$.
(d) Use $\mathbf{v} + (-1)\mathbf{v} = \big(1 + (-1)\big)\mathbf{v} = 0 \cdot \mathbf{v}$ together with (c).

Section 7.2

1. $\mathbf{v}$ is in span $\left\{3x^2 + x - 1,\ x^2 - 3x + 2\right\}$.

3. $\mathbf{v}$ is in span $\left\{3x^2 + x - 1,\ x^2 - 3x + 2\right\}$.

5. $\mathbf{v} = x^3 + 2x^2 - 3x$ is not in span $\left\{x^3 + x - 2,\ x^2 + 2x + 1,\ x^3 - x^2 + x\right\}$.

7. $\mathbf{v} = x^2 + 4x + 4$ is in span $\left\{x^3 + x - 2,\ x^2 + 2x + 1,\ x^3 - x^2 + x\right\}$.

9. $\mathbf{v}$ is in span $\left\{\begin{bmatrix}1 & 2 & 1\\0 & 1 & 3\end{bmatrix}, \begin{bmatrix}0 & 3 & 1\\-1 & 1 & 0\end{bmatrix}\right\}$.

11. $\mathbf{v}$ is in span $\left\{\begin{bmatrix}1 & 2 & 1\\0 & 1 & 3\end{bmatrix}, \begin{bmatrix}0 & 3 & 1\\-1 & 1 & 0\end{bmatrix}\right\}$.

13. $\mathbf{v}$ is in span $\left\{\begin{bmatrix}-1 & 3\\4 & 1\end{bmatrix}, \begin{bmatrix}0 & 2\\5 & -3\end{bmatrix}, \begin{bmatrix}1 & 4\\2 & 1\end{bmatrix}\right\}$.

15. $\mathbf{v}$ is in span $\left\{\begin{bmatrix}-1 & 3\\4 & 1\end{bmatrix}, \begin{bmatrix}0 & 2\\5 & -3\end{bmatrix}, \begin{bmatrix}1 & 4\\2 & 1\end{bmatrix}\right\}$.

17. $\left\{x^2 - 3, 3x^2 + 1\right\}$ is linearly independent in $\mathbf{P}^2$.

19. $\left\{x^3 + 2x + 4, x^2 - x - 1, x^3 + 2x^2 + 2\right\}$ is not linearly independent in $\mathbf{P}^3$.

21. $\left\{\begin{bmatrix}2 & -1\\1 & 3\end{bmatrix}, \begin{bmatrix}-4 & 2\\-2 & -6\end{bmatrix}\right\}$ is not linearly independent in $\mathbf{R}^{2\times 2}$.

23. $\left\{\begin{bmatrix}1 & 0 & 1\\2 & 1 & 4\end{bmatrix}, \begin{bmatrix}3 & 1 & 2\\0 & 3 & 3\end{bmatrix}\right\}$ is linearly independent in $\mathbf{R}^{2\times 3}$.

25. $\left\{\sin^2(x), \cos^2(x), 1\right\}$ is not linearly independent in $C[0, \pi]$.

27. For example,
$$\left\{\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\right\}$$
spans $\mathbf{R}^{2\times 2}$ but is not linearly independent.

29. For example, let $V = \mathbf{P}$, then $\left\{1, x, x^2, x^3, \dots\right\}$ is an infinite linearly independent subset.

31. Let $\mathcal{V}_1 = \{(1, 0, 0, \dots), (0, 0, 1, 0, 0 \dots), (0, 0, 0, 0, 1, 0, 0, \dots)\}$ and $\mathcal{V}_2 = \{(0, 1, 0, 0, \dots), (0, 0, 0, 1, 0, 0 \dots), (0, 0, 0, 0, 0, 1, 0, 0, \dots)\}$. Then $\mathcal{V}_1$ and $\mathcal{V}_2$ are infinite linearly independent subsets of $\mathbf{R}^\infty$ and span $(\mathcal{V}_1) \cap$ span $(\mathcal{V}_2) = \{\mathbf{0}\}$.

33. False

35. False

37. False

39. False

41. True

43. HINT: Show each polynomial is a linear combination of the given set.

45. HINT: See hint given with problem.

47. HINT: Consider cases $\mathbf{v}_1 = \mathbf{0}$ and $\mathbf{v}_1 \neq \mathbf{0}$ separately.

49. HINT: Suppose that $\{\mathbf{v}_1, \dots, \mathbf{v}_m\}$ spans $\mathbf{R}^\infty$. Truncate each vector to the first $m+1$ components. Then the new vectors must also span $\mathbf{R}^{m+1}$, but cannot.

51. HINT: Apply Theorem 7.9(a).

53. HINT: $\mathbf{v}$ is a linear combination of $\{\mathbf{v}_1, \dots, \mathbf{v}_m\}$.

55. $\{x, \sin(\pi x/2), e^x\}$ is linearly independent.

57. $\{e^x, \cos^2(x), \cos(2x), 1\}$ is linearly dependent, so method shown in Example 9 will not work.

Section 7.3

1. $\mathcal{V}$ has too few vectors to be a basis for $\mathbf{P}^2$.

3. $\mathcal{V}$ could be a basis for $\mathbf{R}^{2\times 2}$, since $\dim\left(\mathbf{R}^{2\times 2}\right) = 4$ and $\mathcal{V}$ has 4 vectors.

5. $\mathcal{V}$ has too few vectors to be a basis for $\mathbf{P}^4$.

7. $\mathcal{V}$ is a basis.

9. $\mathcal{V}$ is not a basis.

11. $\mathcal{V}$ is not a basis.

13. dim $(S) = 8$, and a basis for S is

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

15. dim $(S) = 3$, and a basis for S is

$$\left\{ \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix} \right\}$$

17. HINT: S is equivalent to the set of 2×2 matrices A such that $A\mathbf{v} = \mathbf{0}$.

19. dim $(S) = \infty$. For example,

$$\{x(x-1)(x-2),\ x^2(x-1)(x-2),\ x^3(x-1)(x-2), \ldots\}$$

is an infinite set of linearly independent vectors in $C(\mathbf{R})$, each of which vanishes at $k = 0, 1, 2$.

21. We extend $\mathcal{V}$ to $\left\{2x^2+1, 4x-3, 1\right\}$ to obtain a basis for $\mathbf{P}^2$.

23. We extend $\mathcal{V}$ to

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$$

to obtain a basis for $\mathbf{R}^{2\times 2}$.

25. We reduce the set $\mathcal{V}$ to $\{x+1, x+2\}$ to obtain a basis for $\mathbf{P}^1$.

27. HINT: Show that $\{\cos(t), \sin(t)\}$ is a basis for S.

29. A basis for S is the set $\{1, x\}$.

31. For example, let $V = \mathbf{P}$, and $S = \text{span}\{1\}$.

33. For example, let $V = \mathbf{P}^1$, and $S = \text{span}\{1\}$.

35. For example, let $V = \mathbf{P}$, and let $S = \text{span}\left\{1, x^2, x^4, x^6, \ldots\right\}$.

37. False

39. False

41. True

43. False

45. False

47. HINT: It is enough to show that $\{\mathbf{v}_1, 2\mathbf{v}_2, \ldots, k\mathbf{v}_k\}$ is linearly independent.

49. HINT: Show that the set is linearly independent and spans $\mathbf{R}^{2\times 2}$.

51. HINT: Start with a basis for V, and remove one vector at a time to obtain a basis for each of $S_{m-1}, S_{m-2}, \ldots$.

53. HINT: See hint given with problem.

55. HINT: For part (a), show that a basis for V_1 must also be a basis for V_2.

57. HINT: See proof of corresponding theorem in Section 4.2.

Chapter 8

Section 8.1

1. **(a)** $\mathbf{u}_1 \cdot \mathbf{u}_5 = -3$
(b) $\mathbf{u}_3 \cdot (-3\mathbf{u}_2) = -3$
(c) $\mathbf{u}_4 \cdot \mathbf{u}_7 = 11$
(d) $2\mathbf{u}_4 \cdot \mathbf{u}_7 = 22$

3. **(a)** $\|\mathbf{u}_7\| = \sqrt{29}$
(b) $\|-\mathbf{u}_7\| = \sqrt{29}$
(c) $\|2\mathbf{u}_5\| = 2\sqrt{6}$
(d) $\|-3\mathbf{u}_5\| = 6\sqrt{2}$

5. **(a)** $\|\mathbf{u}_1 - \mathbf{u}_2\| = \sqrt{17}$
(b) $\|\mathbf{u}_3 - \mathbf{u}_8\| = \sqrt{26}$
(c) $\|2\mathbf{u}_6 - (-\mathbf{u}_3)\| = 7$
(d) $\|-3\mathbf{u}_2 - 2\mathbf{u}_5\| = 3\sqrt{11}$

7. **(a)** $\mathbf{u}_1 \cdot \mathbf{u}_3 = -8 \neq 0$, so $\mathbf{u}_1$ and $\mathbf{u}_3$ are not orthogonal.
(b) $\mathbf{u}_3 \cdot \mathbf{u}_4 = 0$, so $\mathbf{u}_3$ and $\mathbf{u}_4$ are orthogonal.
(c) $\mathbf{u}_2 \cdot \mathbf{u}_5 = 4 \neq 0$, so $\mathbf{u}_2$ and $\mathbf{u}_5$ are not orthogonal.
(d) $\mathbf{u}_1 \cdot \mathbf{u}_8 = 8 \neq 0$, so $\mathbf{u}_1$ and $\mathbf{u}_8$ are not orthogonal.

9. $a = \frac{3}{2}$

11. $a = \frac{28}{3}$

13. Set is not orthogonal.

15. Set is not orthogonal.

17. $a = -10$

19. $a = 7$ and $b = 11$

21. $\|\mathbf{u}_1\|^2 = 10$, $\|\mathbf{u}_2\|^2 = 10$, $\|\mathbf{u}_1 + \mathbf{u}_2\|^2 = 20$

23. $\|\mathbf{u}_1\|^2 = 14$, $\|\mathbf{u}_2\|^2 = 26$, $\|\mathbf{u}_1 + \mathbf{u}_2\|^2 = 40$

25. $\|3\mathbf{u}_1 + 4\mathbf{u}_2\| = 2\sqrt{109}$

27. $\mathbf{u}$ is not orthogonal to S.

29. A basis for $S^\perp$ is $\left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}$.

31. A basis for $S^\perp$ is $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$.

33. Let $\mathbf{s}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{s}_2 = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$. Then $\mathbf{s} = \frac{3}{2}\mathbf{s}_1 - \frac{1}{2}\mathbf{s}_2$.

35. For example, $\mathbf{u} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

37. For example, $\mathbf{u} = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$.

39. For example, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.

41. For example, $S = \text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right\}$.

43. For example, $\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right\}$.

45. False

47. False

49. True

51. False

53. True

55. False

57. HINT: Every vector $\mathbf{s}$ in S is a linear combination of a spanning set $\mathcal{S}$.

59. HINT: Show that $\mathbf{e}_i \cdot \mathbf{e}_j = 0$ whenever $i \neq j$.

61. HINT: Apply Theorem 8.2(c) twice.

63. HINT: Use the properties of Theorem 8.2.

65. HINT: Apply equation (2) that follows Definition 8.3.

67. HINT: Suppose that $\mathbf{v}$ is in both S and $S^{\perp}$. Use this to show that $\mathbf{v} \cdot \mathbf{v} = 0$.

69. HINT: If $\mathbf{a}$ in $\mathbf{R}^n$ is a column of A and $\mathbf{x} = (x_1, \ldots, x_n)$, then $\mathbf{a}^T\mathbf{x} = \mathbf{a} \cdot \mathbf{x}$.

71. HINTS: (a) Compare definitions of $\mathbf{u} \cdot \mathbf{v}$ and $\mathbf{u}^T\mathbf{v}$.
(b) Start with $(A\mathbf{u}) \cdot \mathbf{v} = (A\mathbf{u})^T\,\mathbf{v}$.

73. **(a)** $\mathbf{u}_2 \cdot \mathbf{u}_3 = -4$
(b) $\|\mathbf{u}_1\| = \sqrt{39}$
(c) $\|2\mathbf{u}_1 + 5\mathbf{u}_3\| = \sqrt{826}$
(d) $\|3\mathbf{u}_1 - 4\mathbf{u}_2 - \mathbf{u}_3\| = \sqrt{1879}$

75. $\left\{\begin{bmatrix} -5 \\ -7 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{9}{2} \\ -4 \\ 0 \\ 1 \end{bmatrix}\right\}$

Section 8.2

1. **(a)** $\text{proj}_{\mathbf{u}_3}\mathbf{u}_2 = \begin{bmatrix} \frac{2}{5} \\ 0 \\ -\frac{1}{5} \end{bmatrix}$

(b) $\text{proj}_{\mathbf{u}_1}\mathbf{u}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

3. $\text{proj}_S\mathbf{u}_2 = \begin{bmatrix} \frac{2}{5} \\ 0 \\ -\frac{1}{5} \end{bmatrix}$

5. **(a)** $\dfrac{1}{\|\mathbf{u}_1\|}\mathbf{u}_1 = \begin{bmatrix} -\frac{3}{14}\sqrt{14} \\ \frac{1}{14}\sqrt{14} \\ \frac{1}{7}\sqrt{14} \end{bmatrix}$

(b) $\dfrac{1}{\|\mathbf{u}_4\|}\mathbf{u}_4 = k\begin{bmatrix} \frac{1}{14}\sqrt{14} \\ -\frac{3}{14}\sqrt{14} \\ \frac{1}{7}\sqrt{14} \end{bmatrix}$

7. An orthogonal basis for S is
$\left\{\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix}\right\}$

9. An orthogonal basis for S is $\left\{\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix}\right\}$.

11. An orthogonal basis for S is
$\left\{\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 2 \\ -1 \end{bmatrix}\right\}$.

13. An orthogonal basis for S is
$\left\{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \\ 3 \end{bmatrix}\right\}$.

15. $\text{proj}_S\mathbf{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

17. $\text{proj}_S\mathbf{u} = \begin{bmatrix} -\frac{3}{29} \\ -\frac{4}{29} \\ \frac{2}{29} \end{bmatrix}$

19. $\text{proj}_S\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

21. $\text{proj}_S\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

23. An orthonormal basis for S is $\left\{\begin{bmatrix} \frac{1}{10}\sqrt{10} \\ \frac{3}{10}\sqrt{10} \end{bmatrix}, \begin{bmatrix} \frac{3}{10}\sqrt{10} \\ -\frac{1}{10}\sqrt{10} \end{bmatrix}\right\}$.

25. An orthonormal basis for S is $\left\{\begin{bmatrix} -\frac{2}{3} \\ \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}, \begin{bmatrix} \frac{3}{29}\sqrt{29} \\ \frac{4}{29}\sqrt{29} \\ -\frac{2}{29}\sqrt{29} \end{bmatrix}\right\}$.

27. An orthonormal basis for S is $\left\{\begin{bmatrix} \frac{1}{3}\sqrt{3} \\ -\frac{1}{3}\sqrt{3} \\ 0 \\ \frac{1}{3}\sqrt{3} \end{bmatrix}, \begin{bmatrix} \frac{1}{2}\sqrt{2} \\ \frac{1}{3}\sqrt{2} \\ \frac{1}{3}\sqrt{2} \\ -\frac{1}{6}\sqrt{2} \end{bmatrix}\right\}$.

29. An orthonormal basis for S is

$$\left\{ \begin{bmatrix} -\frac{1}{2}\sqrt{2} \\ 0 \\ \frac{1}{2}\sqrt{2} \end{bmatrix}, \begin{bmatrix} \frac{1}{6}\sqrt{6} \\ \frac{1}{3}\sqrt{6} \\ \frac{1}{6}\sqrt{6} \end{bmatrix}, \begin{bmatrix} \frac{1}{3}\sqrt{3} \\ -\frac{1}{3}\sqrt{3} \\ -\frac{1}{3}\sqrt{3} \end{bmatrix} \right\}.$$

31. For example, let $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

33. For example, let $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

35. For example, let $\mathbf{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

37. True

39. True

41. True

43. True

45. False

47. HINTS:

(**a**) Show that S_i is a subset of S_j for $i < j$.

(**b**) Reverse the hint for (a).

49. HINT: Show that $\mathbf{u} \cdot (\mathbf{u} + \mathbf{v}) \neq 0$ and $\mathbf{v} \cdot (\mathbf{u} + \mathbf{v}) \neq 0$.

51. HINT: Show the two required properties of a linear transformation hold.

55. HINT: Recall that $\text{proj}_S \mathbf{u}$ and $\mathbf{u} - \text{proj}_S \mathbf{u}$ are orthogonal and use the Pythagorean theorem.

57. HINTS:

(**a**) Use the hint given with this part of the problem.

(**b**) $\|\text{proj}_{\mathbf{v}} \mathbf{u}\| = \frac{|\mathbf{u} \cdot \mathbf{v}|}{\|\mathbf{v}\|}$.

(**c**) Show $\|\text{proj}_{\mathbf{v}} \mathbf{u}\| = \|\mathbf{u}\|$ only when $\mathbf{u} = c\mathbf{v}$.

59. An orthonormal basis is

$$\left\{ \begin{bmatrix} \frac{1}{22}\sqrt{22} \\ \frac{1}{11}\sqrt{22} \\ -\frac{2}{11}\sqrt{22} \\ -\frac{1}{22}\sqrt{22} \end{bmatrix}, \begin{bmatrix} -\frac{9}{1738}\sqrt{22}\sqrt{395} \\ \frac{21}{4345}\sqrt{22}\sqrt{395} \\ \frac{13}{4345}\sqrt{22}\sqrt{395} \\ -\frac{13}{1738}\sqrt{22}\sqrt{395} \end{bmatrix}, \begin{bmatrix} \frac{86}{69757}\sqrt{395}\sqrt{883} \\ \frac{929}{1046355}\sqrt{395}\sqrt{883} \\ \frac{782}{1046355}\sqrt{395}\sqrt{883} \\ \frac{4}{209271}\sqrt{395}\sqrt{883} \end{bmatrix} \right\}$$

61. $\text{proj}_S \mathbf{u} = \begin{bmatrix} \frac{2902}{1477} \\ -\frac{1713}{2954} \\ -\frac{1447}{1477} \\ \frac{1903}{422} \end{bmatrix}$

Section 8.3

1. Not symmetric

3. Symmetric

5. Not symmetric

7. Not symmetric

9. Not orthogonal

11. Orthogonal

13. Not orthogonal

15. $P = \begin{bmatrix} \frac{1}{5}\sqrt{5} & -\frac{2}{5}\sqrt{5} \\ \frac{2}{5}\sqrt{5} & \frac{1}{5}\sqrt{5} \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$

17. $P = \begin{bmatrix} \frac{1}{3}\sqrt{3} & \frac{1}{2}\sqrt{2} & -\frac{1}{6}\sqrt{6} \\ \frac{1}{3}\sqrt{3} & -\frac{1}{2}\sqrt{2} & -\frac{1}{6}\sqrt{6} \\ \frac{1}{3}\sqrt{3} & 0 & \frac{1}{3}\sqrt{6} \end{bmatrix}$, $D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

19. $P = \begin{bmatrix} -\frac{1}{5}\sqrt{5} & \frac{2}{5}\sqrt{5} \\ \frac{2}{5}\sqrt{5} & \frac{1}{5}\sqrt{5} \end{bmatrix}$, $D = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$

21. $P = \begin{bmatrix} -\frac{1}{2}\sqrt{2} & \frac{1}{6}\sqrt{6} & \frac{1}{3}\sqrt{3} \\ 0 & -\frac{1}{3}\sqrt{6} & \frac{1}{3}\sqrt{3} \\ \frac{1}{2}\sqrt{2} & \frac{1}{6}\sqrt{6} & \frac{1}{3}\sqrt{3} \end{bmatrix}$, $D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

23. $P = \begin{bmatrix} -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix}$, $D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

25. $\lambda_1 = 1$ and $\lambda_2 = 11$.

27. $\lambda_1 = 0$, $\lambda_2 = 1$, and $\lambda_3 = 5$

29. $Q^{-1} = \begin{bmatrix} \frac{1}{5} & -\frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} \end{bmatrix}$

31. $Q^{-1} = \begin{bmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

33. $Q = \begin{bmatrix} \frac{3}{13}\sqrt{13} & -\frac{2}{13}\sqrt{13} \\ \frac{2}{13}\sqrt{13} & \frac{3}{13}\sqrt{13} \end{bmatrix}$, $R = \begin{bmatrix} \sqrt{13} & 0 \\ 0 & \sqrt{13} \end{bmatrix}$

35. $Q = \begin{bmatrix} \frac{1}{10}\sqrt{10} & \frac{3}{10}\sqrt{10} \\ \frac{3}{10}\sqrt{10} & -\frac{1}{10}\sqrt{10} \end{bmatrix}$, $R = \begin{bmatrix} \sqrt{10} & \sqrt{10} \\ 0 & \sqrt{10} \end{bmatrix}$.

37. $Q = \begin{bmatrix} \frac{1}{9}\sqrt{3} & \frac{25}{1629}\sqrt{1086} \\ \frac{1}{9}\sqrt{3} & -\frac{85}{3258}\sqrt{1086} \\ \frac{5}{9}\sqrt{3} & \frac{7}{3258}\sqrt{1086} \end{bmatrix}$, $R = \begin{bmatrix} 3\sqrt{3} & \frac{4}{9}\sqrt{3} \\ 0 & \frac{1}{9}\sqrt{1086} \end{bmatrix}$

39. $Q = \begin{bmatrix} -\frac{2}{3} & \frac{3}{29}\sqrt{29} \\ \frac{2}{3} & \frac{4}{29}\sqrt{29} \\ \frac{1}{3} & -\frac{2}{29}\sqrt{29} \end{bmatrix}$, $R = \begin{bmatrix} 3 & 0 \\ 0 & \sqrt{29} \end{bmatrix}$

41. For example, $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

43. For example, $A = \begin{bmatrix} \frac{7}{5} & -\frac{6}{5} \\ -\frac{6}{5} & -\frac{2}{5} \end{bmatrix}$.

45. For example, $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

47. For example, $A = \begin{bmatrix} -2 & 2 \\ -6 & 5 \end{bmatrix}$ (Example 1, Section 6.4).

49. True

51. True

53. True

55. False

57. True

59. HINT: $A^TA = I$.

61. For vectors $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u}^T\mathbf{v} = \mathbf{u}\cdot\mathbf{v}$.

63. HINT: How is A related to A^T?

65. HINT: Show that A^2 is symmetric.

67. $D \approx \begin{bmatrix} 8.0463 & 0 & 0 \\ 0 & 2.2795 & 0 \\ 0 & 0 & -3.3258 \end{bmatrix}$

$P \approx \begin{bmatrix} 0.3603 & -0.8287 & -0.4282 \\ -0.3790 & -0.5495 & 0.7446 \\ 0.8524 & 0.1060 & 0.5120 \end{bmatrix}$

69. $D \approx \begin{bmatrix} 7.624 & 0 & 0 & 0 \\ 0 & -1.211 & 0 & 0 \\ 0 & 0 & 5.639 & 0 \\ 0 & 0 & 0 & -6.051 \end{bmatrix}$

$P \approx \begin{bmatrix} -0.1376 & -0.6216 & 0.7118 & 0.2968 \\ -0.7426 & 0.5164 & 0.1391 & 0.4038 \\ -0.0078 & 0.3699 & 0.6127 & -0.6984 \\ 0.6558 & 0.4585 & 0.3140 & 0.5110 \end{bmatrix}$

71. $Q = \begin{bmatrix} -\frac{1}{2}\sqrt{2} & \frac{1}{3}\sqrt{3} & \frac{1}{6}\sqrt{6} \\ 0 & \frac{1}{3}\sqrt{3} & -\frac{1}{3}\sqrt{6} \\ \frac{1}{2}\sqrt{2} & \frac{1}{3}\sqrt{3} & \frac{1}{6}\sqrt{6} \end{bmatrix}$,

$R = \begin{bmatrix} \sqrt{2} & -\sqrt{2} & \sqrt{2} \\ 0 & 2\sqrt{3} & 4\sqrt{3} \\ 0 & 0 & 0 \end{bmatrix}$

73. $Q = \begin{bmatrix} \frac{1}{2} & \frac{1}{70}\sqrt{35} & \frac{47}{6790}\sqrt{6790} \\ \frac{1}{2} & \frac{9}{70}\sqrt{35} & -\frac{16}{3395}\sqrt{6790} \\ \frac{1}{2} & -\frac{3}{70}\sqrt{35} & \frac{17}{3395}\sqrt{6790} \\ -\frac{1}{2} & \frac{1}{10}\sqrt{35} & \frac{7}{970}\sqrt{6790} \end{bmatrix}$,

$R = \begin{bmatrix} 2 & \frac{3}{2} & 2 \\ 0 & \frac{1}{2}\sqrt{35} & \frac{22}{35}\sqrt{35} \\ 0 & 0 & \frac{2}{35}\sqrt{6790} \end{bmatrix}$

Section 8.4

1. $\sigma_1 = \sqrt{8}$ and $\sigma_2 = \sqrt{2}$

3. $\sigma_1 = \sqrt{16} = 4$ and $\sigma_2 = \sqrt{4} = 2$

5. $\sigma_1 = 3$ and $\sigma_2 = \sqrt{5}$

7. $\sigma_1 = \sqrt{4+\sqrt{5}} \approx 2.497$ and $\sigma_2 = \sqrt{4-\sqrt{5}} \approx 1.328$

9. $V = \begin{bmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{bmatrix}$, $\Sigma = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$,

$U = \begin{bmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \end{bmatrix}$

11. $V \approx \begin{bmatrix} 0.2298 & 0.9732 \\ -0.9732 & 0.2298 \end{bmatrix}$, $\Sigma \approx \begin{bmatrix} 3.199 & 0 \\ 0 & 2.401 \\ 0 & 0 \end{bmatrix}$,

$U \approx \begin{bmatrix} -0.1606 & 0.9064 & -0.3906 \\ -0.9845 & -0.1182 & 0.1302 \\ 0.07183 & 0.4053 & 0.9113 \end{bmatrix}$

13. $A = V\Sigma^TU^T$, where $V = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 3 & 0 \\ 0 & \sqrt{2} \\ 0 & 0 \end{bmatrix}$,

$U = \begin{bmatrix} \frac{2}{3} & -\frac{1}{2}\sqrt{2} & -\frac{1}{6}\sqrt{2} \\ \frac{2}{3} & \frac{1}{2}\sqrt{2} & -\frac{1}{6}\sqrt{2} \\ \frac{1}{3} & 0 & \frac{2}{3}\sqrt{2} \end{bmatrix}$

15. $A = V\Sigma^TU^T$, where $V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 3 & 0 \\ 0 & \sqrt{3} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$,

$U = \begin{bmatrix} \frac{2}{3} & \frac{1}{3}\sqrt{3} & -\frac{1}{6}\sqrt{2} & -\frac{1}{6}\sqrt{6} \\ \frac{2}{3} & -\frac{1}{3}\sqrt{3} & -\frac{1}{6}\sqrt{2} & \frac{1}{6}\sqrt{6} \\ \frac{1}{3} & 0 & \frac{2}{3}\sqrt{2} & 0 \\ 0 & \frac{1}{3}\sqrt{3} & 0 & \frac{1}{3}\sqrt{6} \end{bmatrix}$

17. Numerical rank of A is 2.

19. Numerical rank of A is 2.

21. False

23. True

25. True

27. $\sigma_1\mathbf{u}_1\mathbf{v}_1^T = \begin{bmatrix} -0.1181 & 0.5000 \\ -0.7237 & 3.065 \\ 0.0528 & -0.2236 \end{bmatrix}$

$\sigma_1\mathbf{u}_1\mathbf{v}_1^T + \sigma_2\mathbf{u}_2\mathbf{v}_2^T = \begin{bmatrix} 2.000 & 1.0 \\ -0.9999 & 3.000 \\ 0.9998 & 2.419\times 10^{-5} \end{bmatrix}$

29. $\sigma_1\mathbf{u}_1\mathbf{v}_1^T = \begin{bmatrix} 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\sigma_1\mathbf{u}_1\mathbf{v}_1^T + \sigma_2\mathbf{u}_2\mathbf{v}_2^T = \begin{bmatrix} 2 & 2 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix}$

31. HINT: If $A = U\Sigma V^T$, then $A^T = V\Sigma^TU^T$. Compare the nonzero terms of Σ and Σ^T.

33. HINT: Since U and V are orthogonal, $U^{-1} = U^T$ and $V^{-1} = V^T$.

35. HINT: Simplify $(PA)^TPA$, using P orthogonal.

37. HINTS:

(a) Note that $A^TA\mathbf{x} = A^T(A\mathbf{x})$.

(b) Recall that $\left(\text{col}(A)\right)^{\perp} = \text{null}(A^T)$.

(c) Show that $\text{null}(A)$ and $\text{null}(A^TA)$ are subsets of each other.

39. $V \approx \begin{bmatrix} 0.4527 & 0.8916 \\ 0.8916 & -0.4528 \end{bmatrix}$, $\Sigma \approx \begin{bmatrix} 5.9667 & 0 \\ 0 & 1.8436 \end{bmatrix}$,

$U \approx \begin{bmatrix} 0.9748 & 0.2228 \\ 0.2230 & -0.9748 \end{bmatrix}$

41. $V \approx \begin{bmatrix} 0.8224 & 0.4739 & 0.3147 \\ -0.2477 & 0.7963 & -0.5519 \\ -0.5121 & 0.3760 & 0.7722 \end{bmatrix}$,

$\Sigma \approx \begin{bmatrix} 5.5371 & 0 & 0 \\ 0 & 4.3320 & 0 \\ 0 & 0 & 3.2518 \end{bmatrix}$,

$U \approx \begin{bmatrix} -0.9276 & -0.3734 & -0.008949 \\ -0.08420 & 0.1860 & 0.9789 \\ -0.3639 & 0.9089 & -0.2039 \end{bmatrix}$

Section 8.5

1. $\text{proj}_S \mathbf{y} = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$

3. $\text{proj}_S \mathbf{y} = \begin{bmatrix} \frac{199}{189} \\ -\frac{299}{189} \\ \frac{218}{189} \end{bmatrix}$

5. $\begin{aligned} 14x_1 - 3x_2 &= 23 \\ -3x_1 + 6x_2 &= -2 \end{aligned}$

7. $\begin{aligned} 6x_1 + 3x_2 - 2x_3 &= 6 \\ 3x_1 + 18x_2 - 23x_3 &= 27 \\ -2x_1 - 23x_2 + 30x_3 &= -34 \end{aligned}$

9. $x_1 = -\frac{152}{195}$ and $x_2 = -\frac{17}{195}$

11. $x_1 = -2t$, $x_2 = -5t - \frac{4}{3}$, and $x_3 = t$

13. The normal equations are

$$\begin{bmatrix} 2 & 2 & 4 \\ 2 & 4 & 8 \\ 4 & 8 & 16 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 20 \end{bmatrix}.$$

We obtain infinitely many solutions since there are infinitely many parabolas that pass through two given points.

15. For example,

$$\begin{aligned} x_1 \quad &= 0 \\ x_2 &= 0 \\ x_1 + x_2 &= 1 \end{aligned}$$

17. For example,

$$\begin{aligned} x_1 + x_2 &= 0 \\ 2x_1 + 2x_2 &= 0 \\ 3x_1 + 3x_2 &= 0 \\ 4x_1 + 4x_2 &= 1 \end{aligned}$$

19. For example,

$$\begin{aligned} x_1 \quad &= 0 \\ x_2 \quad &= 0 \\ x_3 &= 0 \end{aligned}$$

21. False

23. True

25. True

27. False

29. Hint: Make the columns of A orthonormal—then this is true.

31. Hint: $A^T A$ is an identity matrix.

33. Hint: Use hint given with problem.

35. $y = 2.2071 + 0.5214x$

37. $y = 2.081 + 0.07458x$

39. $y = 1.667 + 0.09x + 0.3833x^2$

41. $y = 2.191 - 0.06x - 0.5857x^2$

43. $y = 0.9975e^{0.9702x}$

45. $y = 15.49e^{-0.1564x}$

47. $y = 2.173x^{1.306}$

49. $y = 38.51x^{-0.5491}$

51. $p = 0.2001d^{1.499}$

53. $f(t) = 2199.8 + 2.65t - 16.75t^2$, $t = 11.539$ seconds to hit the ground.

55. $y \approx 2.307e^{-0.2211t}$. The initial size of the sample is $y \approx 2.307$ grams. The amount present at $t = 15$ is $y \approx 0.08370$ grams.

Chapter 9

Section 9.1

1. $T(\mathbf{v}_2 - 2\mathbf{v}_1) = \begin{bmatrix} -5 \\ -3 \end{bmatrix}$

3. $T\left(2x^2 - 4x - 1\right) = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$

5. Hint: Focus on Definition 9.1 or Theorem 9.2.

7. Hint: Focus on Definition 9.1 or Theorem 9.2.

9. Hint: Focus on Definition 9.1 or Theorem 9.2.

11. T is a linear transformation. Apply Theorem 9.2 to show this.

13. T is a linear transformation. Apply Theorem 9.2 to show this.

15. T is a linear transformation. Apply Theorem 9.2 to show this.

17. T is a linear transformation. Apply Theorem 9.2 to show this.

19. T is a linear transformation. Apply Theorem 9.2 to show this.

21. T is not a linear transformation.

23. $\ker(T) = \{p(x) : p(x) = ax + a\}$, $\text{range}(T) = \mathbf{R}$

25. $\ker(T) = \{\mathbf{0}_{\mathbf{P}^2}\}$

$\text{range}(T) = \text{span}\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

27. T is not one-to-one, but is onto.

29. T is not one-to-one, but is onto.

31. $V = \mathbf{R}$ and $W = \mathbf{R}^2$, and define $T(a) = \left(\begin{bmatrix} a \\ 0 \end{bmatrix}\right)$.

33. $V = \mathbf{R}^2$ and $W = \mathbf{R}^2$, and define $T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} a \\ 0 \end{bmatrix}$.

35. $V = \mathbf{R}^4$ and $W = \mathbf{R}^3$, and define $T\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right) = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

37. $V = \mathbf{R}^k$ and $W = \mathbf{R}$, and define $T(\mathbf{v}) = \mathbf{0}$.

39. True

41. True

43. False

45. True

47. False

49. $\dim\left(\text{range}(T)\right) = 3$

51. HINT: Use property (a) of a linear transformation.

53. HINT: Apply Theorem 9.2

55. HINT: Use $\mathbf{0}_V + \mathbf{0}_V = \mathbf{0}_V$.

57. HINT: Exercise 55 can be used here to show one of the required properties of a subspace.

59. HINT: Use the hint with the problem.

61. HINT: Use the extended version of Theorem 9.2.

63. HINT: Recall that differentiation distributes across sums of functions.

65. HINT: Recall that differentiation distributes across sums of functions.

67. HINT: $(x^2 p(x))' = 2xp(x) + x^2 p'(x)$

Section 9.2

1. $\dim(V) = \dim\left(\mathbf{R}^8\right) = 8$, and $\dim(W) = \dim\left(\mathbf{P}^9\right) = 10$. Since $\dim(V) \neq \dim(W)$, the vector spaces are not isomorphic.

3. $\dim(V) = \dim\left(\mathbf{R}^{3\times 6}\right) = 18$, and $\dim(W) = \dim\left(\mathbf{P}^{17}\right) = 18$. Since $\dim(V) = \dim(W)$, the vector spaces are isomorphic.

5. $\dim(V) = \dim\left(\mathbf{R}^{13}\right) = 13$, and $\dim(W) = \dim(\mathbf{C}[0, 1]) = \infty$. Since $\dim(V) \neq \dim(W)$, the vector spaces are not isomorphic.

7. HINT: A polynomial is identically zero exactly when all of its coefficients are zero. This can be used to show that T is one-to-one. You need also show that T is a linear transformation and is onto.

9. HINT: A matrix is zero exactly when all of its entries are zero. This can be used to show that T is one-to-one. You need also show that T is a linear transformation and is onto.

11. T is an isomorphism.

13. T is not an isomorphism.

15. $T^{-1}\left(\begin{bmatrix} c \\ d \end{bmatrix}\right) = (c/2 + d)\,x + c/2$

17. $T^{-1}(ax^2 + bx + c) = cx^2 - bx + a$, so that $T^{-1} = T$.

19. HINT: Note that all vectors in S have the form $\begin{bmatrix} a_1 \\ a_2 \\ 0 \end{bmatrix}$.

21. HINT: Focus carefully on the form of a general vector in $\mathbf{P}_e$. It is helpful to consider a few concrete cases.

23. $T\left(\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}\right) = ax^4 + bx^3 + cx^2 + dx + e$ is an isomorphism.

25. $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ax^3 + bx^2 + cx + d$ is an isomorphism.

27. $S = \text{span}\left\{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\right\}$

29. Let S be the set of all vectors of the form $(a_1, a_2, \ldots a_n, 0, 0, 0, \ldots)$. That is, all infinite vectors with entries equal to zero from some point on.

31. False

33. True

35. False

37. True

39. False

41. False

43. False

45. HINT: T must be one-to-one and onto to have an inverse.

47. HINT: The proof of Theorem 3.19 can be used as a model for showing that T^{-1} is also a linear transformation.

49. HINT: T is always onto range(T).

Section 9.3

1. $\mathbf{v} = \begin{bmatrix} -11 \\ -4 \end{bmatrix}$

3. $\mathbf{v} = -x^2 - 14x - 9$

5. $\mathbf{v}_{\mathcal{G}} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}_{\mathcal{G}}$

7. $\mathbf{v}_{\mathcal{G}} = \begin{bmatrix} 3 \\ -5 \\ 6 \end{bmatrix}_{\mathcal{G}}$

9. $\mathbf{v} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}_{\mathcal{G}}$

11. $\mathbf{v} = \begin{bmatrix} \frac{21}{4} \\ \frac{9}{2} \\ -\frac{7}{4} \end{bmatrix}_{\mathcal{G}}$

13. $T(\mathbf{v}_{\mathcal{G}}) = \begin{bmatrix} 17 \\ 28 \end{bmatrix}$

15. $T(\mathbf{v}_{\mathcal{G}}) = 27x^2 + 12x + 67$

17. $T(\mathbf{v}_{\mathcal{G}}) = 4\cos x - 9\sin x + 9e^{-x}$

19. $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

21. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

23. $A = \begin{bmatrix} -3 & 0 \\ 2 & 1 \end{bmatrix}$

25. $A = \begin{bmatrix} 7 & 12 \\ -18 & -31 \end{bmatrix}$

27. **(a)** $\begin{bmatrix} a/2 & b/2 & c/2 \\ d & e & f \end{bmatrix}$ **(b)** $\begin{bmatrix} 3a & b & c \\ 3d & e & f \end{bmatrix}$

29. **(a)** $\begin{bmatrix} c & a & b \\ f & d & e \end{bmatrix}$ **(b)** $\begin{bmatrix} d & f & e \\ a & c & b \end{bmatrix}$

31. $T^{-1}(x) = -2x - 4$

33. $T^{-1}(x+1) = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$

35. $\mathbf{v} = 2x^2 - 3$, and $\mathcal{G} = \{x^2, x, 1\}$

37. $\mathcal{G} = \left\{ \begin{bmatrix} -7/3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 5/4 \end{bmatrix} \right\}$

39. $V = \mathbf{R}^3$ and $W = \mathbf{R}^2$, and let $\mathcal{G} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ be the basis for V, and $\mathcal{Q} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ be the basis for W. Define $T(\mathbf{v}) = A\mathbf{v}$, where $A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$.

41. False

43. True

45. True

47. HINT: The general results $[c\mathbf{v}]_{\mathcal{G}} = c[\mathbf{v}]_{\mathcal{G}}$ and $[\mathbf{v}_1 + \mathbf{v}_2]_{\mathcal{G}} = [\mathbf{v}_1]_{\mathcal{G}} + [\mathbf{v}_2]_{\mathcal{G}}$ are useful here.

49. HINT: A more general version of the results given in the answer to Exercise 47 can be used here.

51. HINT: The proof of part (b) follows from induction on n.

Section 9.4

1. $S = \begin{bmatrix} 2 & 5 \\ -1 & 4 \end{bmatrix}$

3. $S = \begin{bmatrix} 3 & 4 & 1 & 0 \\ 2 & 0 & 7 & 6 \\ 1 & 0 & 5 & 2 \\ 0 & 2 & 1 & 3 \end{bmatrix}$

5. $S = \begin{bmatrix} 1 & 9 & -1 \\ -2 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

7. $S = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

9. $A = \begin{bmatrix} 62 & 204 \\ -18 & -59 \end{bmatrix}$

11. $A = \begin{bmatrix} 3 & 6 & 2 \\ -3 & -8 & -3 \\ 10 & 17 & 6 \end{bmatrix}$

13. $A = \begin{bmatrix} -3 & -2 \\ 7 & 5 \end{bmatrix}$

15. $A = \begin{bmatrix} -889 & 1411 \\ -562 & 892 \end{bmatrix}$

17. A and B are not similar matrices.

19. A and B are similar matrices.

21. $V = \mathbf{R}^2$, and let $\mathcal{G} = \left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \end{bmatrix} \right\}$ and $\mathcal{H} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

23. $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $A = \begin{bmatrix} 31 & 12 \\ -67 & -26 \end{bmatrix}$, related by $S = \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}$.

25. True

27. False

29. False

31. False

33. False

35. $D = S_2 S_1$

37. HINT: A and B have the same diagonal matrix D in their diagonalizations.

39. HINT: If S is invertible, then $\left(S^{-1}\right)^T = \left(S^T\right)^{-1}$.

41. A and B are not similar matrices.

43. A and B are similar matrices.

Chapter 10

Section 10.1

1. $\langle \mathbf{u}, \mathbf{v} \rangle = 32$

3. $\langle p, q \rangle = -8$

5. $\langle f, g \rangle = 2$

7. $\langle A, B \rangle = -9$

9. $a = 3$

11. No value of a will make p and q orthogonal.

13. No value of b will make f and g orthogonal.

15. Norm $= \sqrt{33}$

17. Norm $= \sqrt{174}$

19. Norm = $\sqrt{\frac{2}{7}}$

21. $\|A\| = \sqrt{14}$

23. $\text{proj}_{\mathbf{u}}\mathbf{v} = \begin{bmatrix} \frac{32}{15} \\ \frac{64}{15} \\ \frac{32}{15} \end{bmatrix}$

25. $\text{proj}_p q = -\frac{8}{23}x - \frac{16}{69}$

27. $\text{proj}_f g = 0$

29. $\text{proj}_A B = \begin{bmatrix} \frac{1}{3} & -\frac{1}{6} \\ \frac{1}{6} & 0 \end{bmatrix}$

31. $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

33. $\mathbf{u} = \begin{bmatrix} 6/\sqrt{13} \\ -4/\sqrt{13} \end{bmatrix}$, $t_1 = t_2 = 1$

35. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

37. Let $w(x) = \cos(x)$, and define $\langle p, q\rangle = \int_0^1 p(x)\, q(x)\, w(x)\, dx$.

39. $\langle p, q\rangle = 0$ for all p and q in $\mathbf{P}^2$.

41. True

43. True

45. True

47. False

49. False

51. HINT: Use the distributive property of the real numbers to establish (b) and (c) of the definition of inner product.

53. HINT: The solution to Exercise 51 can be used as a model for this problem.

55. HINT: Example 5 can serve as a guide for this proof.

57. HINT: Review the properties of matrix transposes.

59. HINT: $\|c\mathbf{v}\|^2 = \langle c\mathbf{v}, c\mathbf{v}\rangle$

61. HINT: Use induction on k.

63. HINT: $\left\|\frac{1}{\|\mathbf{v}\|}\mathbf{v}\right\| = \left|\frac{1}{\|\mathbf{v}\|}\right| \|\mathbf{v}\|$ by Exercise 59.

65. HINT: Use properties of inner products to verify the required properties of a linear transformation.

67. HINT: $\|\mathbf{u} - \mathbf{v}\|^2 = \langle \mathbf{u} - \mathbf{v}, \mathbf{u} - \mathbf{v}\rangle$

69. HINT: See Theorem 8.8 in Section 8.1.

71. HINT: Apply Exercise 67.

Section 10.2

1. $\left\{ \begin{bmatrix} \frac{2}{9}\sqrt{3} \\ \frac{5}{18}\sqrt{3} \\ \frac{1}{18}\sqrt{3} \end{bmatrix}, \begin{bmatrix} -\frac{1}{22}\sqrt{66} \\ \frac{1}{33}\sqrt{66} \\ -\frac{1}{11}\sqrt{66} \end{bmatrix}, \begin{bmatrix} \frac{8}{99}\sqrt{33} \\ -\frac{7}{198}\sqrt{33} \\ -\frac{23}{198}\sqrt{33} \end{bmatrix} \right\}$

3. $a = -5$, $\left\{ \begin{bmatrix} \frac{1}{3}\sqrt{3} \\ -\frac{1}{6}\sqrt{3} \\ \frac{1}{6}\sqrt{3} \end{bmatrix}, \begin{bmatrix} \frac{1}{3} \\ \frac{1}{6} \\ -\frac{5}{6} \end{bmatrix}, \begin{bmatrix} \frac{1}{6}\sqrt{2} \\ \frac{1}{3}\sqrt{2} \\ \frac{1}{3}\sqrt{2} \end{bmatrix} \right\}$

5. $a = -1$, $\left\{\frac{1}{6}x^2 + \frac{1}{6}x, \frac{1}{2}x^2 - \frac{1}{2}x - 1, \frac{1}{3}x^2 - \frac{2}{3}x\right\}$

7. $\mathbf{v} = (5)\begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix} + (1)\begin{bmatrix} -3 \\ 2 \\ -6 \end{bmatrix} + (-1)\begin{bmatrix} 16 \\ -7 \\ -23 \end{bmatrix}$

9. $\text{proj}_S\mathbf{v} = \begin{bmatrix} \frac{7}{27} \\ \frac{35}{108} \\ \frac{7}{108} \end{bmatrix}$

11. $\text{proj}_S f = 2\sin(x)$

13. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{6}{5} \\ \frac{4}{5} \\ 1 \end{bmatrix} \right\}$

15. $\left\{1, x^2 - \frac{1}{3}\right\}$

17. $\left\{x, -\frac{1}{5}x + 1\right\}$

19. For example, let $\mathbf{u}_2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$.

21. For example, let $p_2(x) = 5x - 3$.

23. For example, $f_1(x) = 0$, $f_2(x) = 1$, and $f_3(x) = \cos(x)$.

25. False

27. True

29. False

31. True

33. HINT: Apply hint given with problem.

35. HINT: The suggested approach works well.

37. HINT: $\text{proj}_S\mathbf{u}$ is in S.

39. **(a)** $\mathbf{u}$

(b) $\mathbf{0}$

41. HINT: Use $\|\mathbf{v}\|^2 = \langle \mathbf{v}, \mathbf{v}\rangle$ and Theorem 10.12.

Section 10.3

1. $y = \frac{5}{2} + 1x$

3. The slope of ℓ_1 would be greater than the slope of ℓ_2.

5. The resulting line will be the same.

7. $f_2(x) = \frac{1}{2} + \frac{2}{\pi}\cos(x)$

9. $f_2(x) = \frac{1}{2} + \frac{2}{\pi}\sin(x)$

11. $f_2(x) = 1 + 2\sin(x) - \sin(2x)$

13. $f_2(x) = \frac{1}{3}\pi^2 - 4\cos(x) + \cos(2x)$

15. $a_2 = 1$ and $b_3 = -1$, with all other coefficients zero.

17. $a_0 = \frac{3}{2}$ and $a_8 = -\frac{1}{2}$, with all other Fourier coefficients zero.

19. $g_1(x) = \frac{3}{2} - \cos(x)$

21. $g_1(x) = \frac{1}{2} + \frac{3}{2}\cos(x) + \frac{3}{2}\sin(x)$

23. For example, consider the data set $\{(-1,-1),(0,1),(1,-1)\}$. This set has ordinary least squares regression line $y = 0$, and weighted least squares regression line with triple the weight on the right-most point $y = -\frac{2}{5} - \frac{1}{2}x$.

25. For example, let $f(x) = 1$. Then, $a_0 = 1$, and all other Fourier coefficients are zero.

27. $f(x) = 1 + x$

29. True

31. False

33. True

35. HINT: $\langle 1, \sin(kx)\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin(kx)\, dx$

37. HINT: $\|\sin(kx)\|^2 = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin^2(kx)\, dx$

39. HINT: Take $u = x$ and $dv = \cos(kx)dx$ in the integration by parts formula.

41. HINT: $\cos(k\pi) = (-1)^k$ and $\sin(k\pi) = 0$ for all integers k.

43. $g_5(x) \approx 2.7125 + 0.5445\cos(x) + 0.05\cos(2x) + 0.1555\cos(3x) + 0.075\cos(4x) + 0.1555\cos(5x) - 0.06768\sin(x) - 0.025\sin(2x) + 0.03232\sin(3x) - 0.03232\sin(5x)$

45. $f_3(x) \approx 3.6761 - 3.6761\cos(x) + 1.4704\cos(2x) - 0.7352\cos(3x) + 3.6761\sin(x) - 2.9409\sin(2x) + 2.2056\sin(3x)$

Chapter 11

Section 11.1

1. $Q(\mathbf{x}_0) = 22$

3. $Q(\mathbf{x}_0) = 8$

5. $Q(\mathbf{x}) = 4x_1^2 + x_2^2$

7. $Q(\mathbf{x}) = x_1^2 + 2x_2^2 + 6x_1x_2$

9. $Q(\mathbf{x}) = x_1^2 + 3x_2^2 - 2x_3^2$

11. $Q(\mathbf{x}) = 2x_1^2 + x_2^2 + 2x_3^2 + 3x_4^2$

13. $A = \begin{bmatrix} 1 & 3 \\ 3 & -5 \end{bmatrix}$

15. $A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 1 & 0 \\ 3 & 0 & -1 \end{bmatrix}$

17. $A = \begin{bmatrix} 5 & 0 & 3 \\ 0 & -1 & -6 \\ 3 & -6 & 3 \end{bmatrix}$

19. A is indefinite.

21. A is indefinite.

23. A is indefinite.

25. A is indefinite.

27. $Q(\mathbf{x}) = x_1^2 + x_2^2$, and $c = -1$

29. For example, let $Q(\mathbf{x}) = x_1^2 - x_2^2$, and $c = 0$. Then $Q(\mathbf{x}) = x_1^2 - x_2^2 = 0 \quad \Rightarrow \quad (x_1 - x_2)(x_1 + x_2) = 0$, and the graph consists of the intersecting lines $x_1 - x_2 = 0$ and $x_1 + x_2 = 0$.

31. The only quadratic form which is also a linear transformation is $Q(\mathbf{x}) = 0$ for all $\mathbf{x}$.

33. $Q(\mathbf{x}) = x_1^2 + x_2^2$

35. True

37. False

39. True

41. HINT: What form must such a quadratic form take?

43. $Q(\mathbf{x}) = \mathbf{x}^T I \mathbf{x} = \mathbf{x}^T\mathbf{x} = \|\mathbf{x}\|^2$, so the identity matrix I_n is the matrix of $Q(\mathbf{x}) = \|\mathbf{x}\|^2$.

45. HINT: Evaluate $\mathbf{0}^T A\mathbf{0}$.

Section 11.2

1. $A_1 = [3]$, $A_2 = \begin{bmatrix} 3 & 5 \\ 5 & 7 \end{bmatrix}$

3. $A_1 = [1]$, $A_2 = \begin{bmatrix} 1 & 4 \\ 4 & 0 \end{bmatrix}$, $A_3 = \begin{bmatrix} 1 & 4 & -3 \\ 4 & 0 & 2 \\ -3 & 2 & 5 \end{bmatrix}$

5. $A_1 = [2]$, $A_2 = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, $A_3 = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 4 \\ 0 & 4 & 0 \end{bmatrix}$,

$A_4 = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 1 & 3 & 4 & 1 \\ 0 & 4 & 0 & -2 \\ -1 & 1 & -2 & 1 \end{bmatrix}$

7. $\det(A_1) = 2 > 0$, $\det(A_2) = -5 < 0$, so A is not positive definite.

9. $\det(A_1) = 1 > 0$, $\det(A_2) = 1 > 0$, $\det(A_3) = 1 > 0$, so A is positive definite.

11. $\det(A_1) = 1 > 0$, $\det(A_2) = 1 > 0$, $\det(A_3) = 1 > 0$, $\det(A_4) = 4 > 0$, so A is positive definite.

13. $\det(A_1) = 1 > 0$, $\det(A_2) = 1 > 0$, so A is positive definite.

$L = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

15. $\det(A_1) = 1 > 0$, $\det(A_2) = 1 > 0$, $\det(A_3) = 1 > 0$, so A is positive definite.

$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

17. $\det(A_1) = 1 > 0$, $\det(A_2) = 4 > 0$, so A is positive definite.

$L = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$ $U = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

19. $\det(A_1) = 1 > 0$, $\det(A_2) = 1 > 0$, $\det(A_3) = 1 > 0$, so A is positive definite.

$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

21. $\det(A_1) = 1 > 0$, $\det(A_2) = 1 > 0$, so A is positive definite.

$$L_c = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

23. $\det(A_1) = 1 > 0$, $\det(A_2) = 1 > 0$, $\det(A_3) = 1 > 0$, so A is positive definite.

$$L_c = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

25. $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

27. $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

29. $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

31. False

33. False

35. True

Section 11.3

1. max $= 2$, min $= -3$

3. max $= 3$, min $= -5$

5. max $= 2$, min $= -4$

7. max $= 5$, min $= 0$

9. max $= 1$, min $= -1$

11. max $= 3$, min $= 0$

13. max $= 20$, min $= 0$

15. max $= 100$, min $= -100$

17. max $= 104$, min $= 0$

19. max $= 19 + \sqrt{370}$, min $= 19 - \sqrt{370}$

21. max $= 3$, min $= 1$

23. max $= 1$, min $= 0$

25. $Q(\mathbf{x}) = x_1^2 + 5x_2^2$

27. $Q(\mathbf{x}) = x_1^2 + 6x_2^2$

29. $Q(\mathbf{x}) = -\frac{1}{9}x_1^2 + \frac{4}{9}x_2^2$

31. True

33. False

35. False

37. HINTS: $q_j \le q_k$ for all $k = 1, 2, \ldots, n$, and $q_k \le q_i$ for all $k = 1, 2, \ldots, n$.

39. HINT: $Q(c\mathbf{x}) = (c\mathbf{x})^T A(c\mathbf{x})$

Section 11.4

1. (a) $\begin{bmatrix} 3i \\ -4+i \\ 2-i \end{bmatrix}$ (b) $\begin{bmatrix} 8+i \\ 11-4i \\ 7+12i \end{bmatrix}$ (c) $\begin{bmatrix} -14+i \\ -12+9i \\ -2-21i \end{bmatrix}$

3. c does not exist.

5. Yes.

7. **(a)** $24 - 6i$ **(b)** 44 **(c)** $\sqrt{35}$

9. Divide $\mathbf{u}$ by $\sqrt{39}$, divide $\mathbf{v}$ by $2\sqrt{10}$.

11. **(a)** $\begin{bmatrix} 2+i & 4-3i \\ -3-i & 3+2i \end{bmatrix}$ **(b)** $\begin{bmatrix} -2-3i & 9-12i \\ -13+5i & 4+i \end{bmatrix}$

13. c does not exist.

15. Yes.

17. **(a)** $18 + 2i$ **(b)** $14 - 50i$ **(c)** $\sqrt{42}$

19. Divide A by $\sqrt{29}$, divide C by $2\sqrt{7}$.

21. **(a)** $(2 + 4i) - (4 - 2i)\,x$ **(b)** $9 - (3 + 3i)\,x$

23. c does not exist.

25. $(2 + i) + (3 - 2i)\,x = \left(-\frac{2}{17} - \frac{43}{17}i\right)(1 + ix) + \left(\frac{12}{17} + \frac{20}{17}i\right)(3 - (1 + i)\,x)$

27. **(a)** $\frac{13}{6} + \frac{5}{3}i$
(b) $\frac{13}{3} + \frac{10}{3}i$
(c) $\frac{2}{3}\sqrt{3}$

29. Divide h_1 by $\frac{2}{3}\sqrt{3}$, divide h_2 by $\frac{2}{3}\sqrt{3}$.

31. $\mathbf{u} = 1$, $\mathbf{v} = i$ in $\mathbf{C}$.

33. $\langle \mathbf{u}, \mathbf{v} \rangle = 2u_1\overline{v_1} + \cdots + 2u_n\overline{v_n}$

35. $V = \mathbf{C}$, and $S = \mathbf{R}$

37. True

39. True

41. False

43. HINT: Apply the properties of complex numbers and arithmetic.

45. HINT: Consider property (2).

47. HINT: Combine properties of complex numbers with definition of inner product space.

49. HINT: Apply property (d) of definition of inner product space.

51. HINT: Note that $0 \le \left\| \mathbf{u} - \frac{\langle \mathbf{u}, \mathbf{v} \rangle}{\|\mathbf{v}\|^2}\mathbf{v} \right\|^2$

Section 11.5

1. $A^* = \begin{bmatrix} 1-i & 2+i \\ -3i & 1-4i \end{bmatrix}$

3. $A^* = \begin{bmatrix} 3-i & 1+4i & 2-2i \\ -5i & -8 & 0 \\ 1+i & 6-i & 7i \end{bmatrix}$

5. $A^* = \begin{bmatrix} 1 & -2i & 3 & 4i \\ 2i & 5 & -6i & 1+i \\ 3 & 6i & 7 & 3+2i \\ -4i & 1-i & 3-2i & 11 \end{bmatrix}$

7. A is not Hermitian.

9. A is Hermitian.

11. A is Hermitian.

13. A is normal.

15. A is normal.

17. A is normal.

19. $A = \begin{bmatrix} 0 & i & i \\ i & 0 & i \\ i & i & 0 \end{bmatrix}$

21. $A = \begin{bmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{bmatrix}$

23. True

25. False

27. True

29. True

31. HINT: If A has real entries, then so does A^T.

33. HINT: Apply the result in Exercise 32.

35. HINT: Apply the result in Exercise 32.

37. HINT: A unitary implies that $A^{-1} = A^*$, so $A^* A = I_n$.

39. HINT: Compute $A^* A$ for A normal and upper triangular.