EXERCISES

For Exercises 1–6, let

$$\mathbf{u} = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} -4 \\ 1 \\ 5 \end{bmatrix} \quad \text{and} \quad \mathbf{w} = \begin{bmatrix} 2 \\ -7 \\ -1 \end{bmatrix}$$

1. Compute $\mathbf{u} - \mathbf{v}$.

2. Compute $-5\mathbf{u}$.

3. Compute $\mathbf{w} + 3\mathbf{v}$.

4. Compute $4\mathbf{w} - \mathbf{u}$.

5. Compute $-\mathbf{u} + \mathbf{v} + \mathbf{w}$.

6. Compute $3\mathbf{u} - 2\mathbf{v} + 5\mathbf{w}$.

In Exercises 7–10, express the given vector equation as a system of linear equations.

7. $x_1 \begin{bmatrix} 3 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \end{bmatrix}$

8. $x_1 \begin{bmatrix} -1 \\ 6 \\ -4 \end{bmatrix} + x_2 \begin{bmatrix} 9 \\ -5 \\ 0 \end{bmatrix} = \begin{bmatrix} -7 \\ -11 \\ 3 \end{bmatrix}$

9. $x_1 \begin{bmatrix} -6 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 16 \end{bmatrix}$

10. $x_1 \begin{bmatrix} 2 \\ 7 \\ 8 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 2 \\ 4 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ 1 \\ 6 \\ 1 \end{bmatrix} + x_4 \begin{bmatrix} 4 \\ 5 \\ 7 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 3 \\ 5 \end{bmatrix}$

In Exercises 11–14, express the given system of linear equations as a vector equation.

11.
$$\begin{aligned} 2x_1 + 8x_2 - 4x_3 &= -10 \\ -x_1 - 3x_2 + 5x_3 &= 4 \end{aligned}$$

12.
$$\begin{aligned} -2x_1 + 5x_2 - 10x_3 &= 4 \\ x_1 - 2x_2 + 3x_3 &= -1 \\ 7x_1 - 17x_2 + 34x_3 &= -16 \end{aligned}$$

13.
$$\begin{aligned} x_1 - x_2 - 3x_3 - x_4 &= -1 \\ -2x_1 + 2x_2 + 6x_3 + 2x_4 &= -1 \\ -3x_1 - 3x_2 + 10x_3 &= 5 \end{aligned}$$

14.
$$\begin{aligned} -5x_1 + 9x_2 &= 13 \\ 3x_1 - 5x_2 &= -9 \\ x_1 - 2x_2 &= -2 \end{aligned}$$

In Exercises 15–18, the general solution to a linear system is given. Express this as a linear combination of vectors.

15.
$$\begin{aligned} x_1 &= -4 + 3s_1 \\ x_2 &= s_1 \end{aligned}$$

16.
$$\begin{aligned} x_1 &= 7 - 2s_1 \\ x_2 &= -3 \\ x_3 &= s_1 \end{aligned}$$

17.
$$\begin{aligned} x_1 &= 4 + 6s_1 - 5s_2 \\ x_2 &= s_2 \\ x_3 &= -9 + 3s_1 \\ x_4 &= s_1 \end{aligned}$$

18.
$$\begin{aligned} x_1 &= 1 - 7s_1 + 14s_2 - s_3 \\ x_2 &= s_3 \\ x_3 &= s_2 \\ x_4 &= -12 + s_1 \\ x_5 &= s_1 \end{aligned}$$

In Exercises 19–22, find three different vectors that are a linear combination of the given vectors.

19. $\mathbf{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$

20. $\mathbf{u} = \begin{bmatrix} 7 \\ 1 \\ -13 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}$

21. $\mathbf{u} = \begin{bmatrix} -4 \\ 0 \\ -3 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -2 \\ -1 \\ 5 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 9 \\ 6 \\ 11 \end{bmatrix}$

22. $\mathbf{u} = \begin{bmatrix} 1 \\ 8 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 4 \\ -2 \\ 5 \\ -5 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 9 \\ 9 \\ 0 \\ 1 \end{bmatrix}$

In Exercises 23–26, a vector equation is given with some unknown entries. Find the unknowns.

23. $-3\begin{bmatrix} a \\ 3 \end{bmatrix} + 4\begin{bmatrix} -1 \\ b \end{bmatrix} = \begin{bmatrix} -10 \\ 19 \end{bmatrix}$

24. $4\begin{bmatrix} 4 \\ a \end{bmatrix} + 3\begin{bmatrix} -3 \\ 5 \end{bmatrix} - 2\begin{bmatrix} b \\ 8 \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$

25. $-\begin{bmatrix} -1 \\ a \\ 2 \end{bmatrix} + 2\begin{bmatrix} 3 \\ -2 \\ b \end{bmatrix} = \begin{bmatrix} c \\ -7 \\ 8 \end{bmatrix}$

26. $-\begin{bmatrix} a \\ 4 \\ -2 \\ -1 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 1 \\ b \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ c \\ -3 \\ -6 \end{bmatrix} = \begin{bmatrix} 11 \\ -4 \\ 3 \\ d \end{bmatrix}$

In Exercises 27–30, determine if **b** is a linear combination of the other vectors. If so, write **b** as a linear combination.

27. $\mathbf{a}_1 = \begin{bmatrix} -2 \\ 5 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 7 \\ -3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 8 \\ 9 \end{bmatrix}$

28. $\mathbf{a}_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ -5 \\ -2 \end{bmatrix}$

29. $\mathbf{a}_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 6 \\ 3 \\ -9 \end{bmatrix}$

30. $\mathbf{a}_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ -4 \\ 5 \end{bmatrix}$