Preparation problems for the discussion sections on September 2nd and 4th

1. *For the following systems determine*

(1) *the augmented matrix,*
(2) *an echelon form of the matrix,*
(3) *the reduced echelon form of the matrix,*
(4) *whether the system is consistent,*
(5) *the set of solutions (in parametric form),*
(6) *how many solutions the system has,*
(7) *the geometric interpretation of the set of solutions.*

System A:

$$\begin{aligned} x_2 &= 3 \\ x_1 + 2x_2 &= 4 \end{aligned}$$

System B:

$$\begin{aligned} x_1 + x_2 &= 3 \\ 2x_1 + 2x_2 &= 6 \end{aligned}$$

System C:

$$\begin{aligned} x_1 + x_2 &= 3 \\ 2x_1 + 2x_2 &= 7 \end{aligned}$$

2. *Some questions to check your understanding:*

a) *What is the largest possible number of pivots a* 4×6 *matrix can have? Why?*
b) *What is the largest possible number of pivots a* 6×4 *matrix can have? Why?*
c) *How many solutions does a consistent linear system of 3 equations and 4 unknowns have? Why?*
d) *Suppose the coefficient matrix corresponding to a linear system is* 4×6 *and has 3 pivot columns. How many pivot columns does the augmented matrix have if the linear system is inconsistent?*

3. *Find a parametric description of the set of solutions of:*

$$\begin{aligned} x_1 + 3x_2 - 5x_3 &= 4 \\ x_1 + 4x_2 - 8x_3 &= 7 \\ -3x_1 - 7x_2 + 9x_3 &= -6 \end{aligned}$$

4. *For which values of* h_1 *and* h_2 *is the following system consistent?*

$$\begin{aligned} x_1 &= h_1 \\ x_2 &= 5 \\ x_1 + 2x_2 &= h_2 \end{aligned}$$

5. *Show that the interchange of two rows of a matrix can be accomplished by a finite sequence of elementary row operations of the other two types.*

6. *Let $A = [a_{ij}]_{3\times 4}$, and let $B = [b_{ij}]_{3\times 4}$ be an echelon form of A.*

(1) *Is it true that, if $a_{11} = 0$, then $b_{11} = 0$?*
(2) *Is it true that, if A has a column of zeros, then B also has a column of zeros?*
(3) *Suppose B has a row of zeros. What can you say about rows of A? (Explain.)*
(4) *Suppose we form a new matrix using some columns of A, let's say the first and the third column. What is an echelon form corresponding to this new matrix?*