Final Exam – Practice

MATH 332 — Differential Equations II
Final Exam: Monday, May 5, 2025

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches). Any typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Problem 1. The final exam will be comprehensive, that is, it will cover the material of the whole semester.

(a) Do the practice problems for both midterms.

(b) Retake both midterm exams.

(c) Do the problems below. (Solutions are posted.)

Problem 2. For $t \geqslant 0$ and $x \in [0, 4]$, consider the heat flow problem:
$$\begin{aligned} u_t &= 2u_{xx} + e^{-x/2} \\ u_x(0,t) &= 3 \\ u(4,t) &= -2 \\ u(x,0) &= f(x) \end{aligned}$$
Determine the steady-state solution and spell out equations characterizing the transient solution.

Problem 3. Using a step size of $h = \frac{1}{3}$, discretize the Dirichlet problem
$$\begin{aligned} u_{xx} + u_{yy} &= 0 \\ u(x,0) &= 3 \\ u(x,1) &= 5 \\ u(0,y) &= 1 \\ u(1,y) &= 2 \end{aligned}$$
where $x \in (0,1)$ and $y \in (0,1)$.

Spell out a system of linear equations for the resulting lattice points. Do not solve that system.

(Note that, for the Dirichlet problem as well as for our discretization, it doesn't matter that the boundary conditions aren't well-defined at the corners.)

Problem 4. Consider the Laplace equation $u_{xx} + u_{yy} = 0$ on the polygonal region with vertices $(0,0)$, $(1,0)$, $(1,1)$, $(2,1)$, $(2,2)$, $(0,2)$. Suppose that $u(0,y) = 5$ for $y \in (0,2)$ and that $u(x,y) = 7$ for all other points (x,y) on the boundary of the region. Discretize this Dirichlet problem using a step size of $h = \frac{1}{2}$.

Spell out a system of linear equations for the resulting lattice points. Do not solve that system.

Problem 5. Find all eigenfunctions and eigenvalues of
$$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y(3) = 0.$$

Problem 6. Find the solution $u(x,t)$, for $0 < x < 3$ and $t \geqslant 0$, to the heat conduction problem
$$2u_t = u_{xx}, \quad u_x(0,t) = 0, \ u(3,t) = 0, \quad u(x,0) = 2\cos\left(\frac{\pi x}{2}\right) + 7\cos\left(\frac{3\pi x}{2}\right).$$
Derive your solution using separation of variables (at some step you may refer to the EVP above).

Armin Straub
straub@southalabama.edu