Midterm #2

MATH 332 — Differential Equations II
Friday, Mar 31, 2023

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 28 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (6 points) Derive a recursive description of a power series solution $y(x)$ of the DE $y'' = (3x^2 - 2)y$.

Problem 2. (3 points) Let $y(x)$ be the unique solution to the IVP $y'' = x + 3y^2$, $y(0) = 2$, $y'(0) = 1$.
Determine the first several terms (up to x^3) in the power series of $y(x)$.

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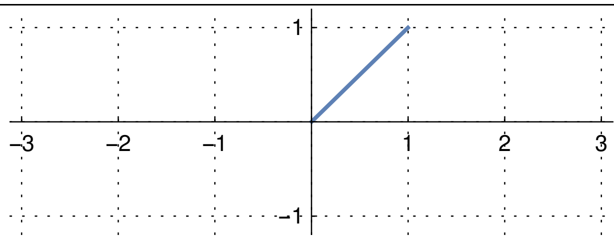
Problem 3. (4 points) Consider the function $f(t) = t$, defined for $t \in [0, 1]$.

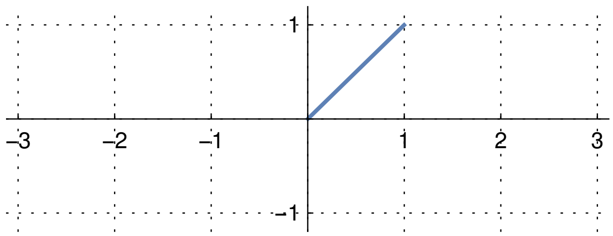
(a) Sketch the Fourier series of $f(t)$ for $t \in [-3, 3]$.

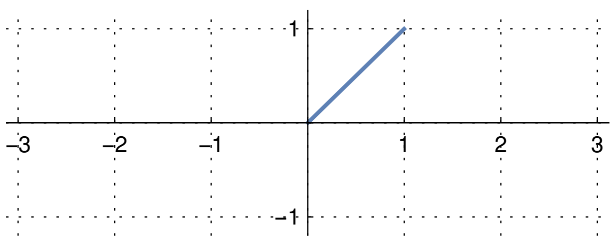
(b) Sketch the Fourier cosine series of $f(t)$ for $t \in [-3, 3]$.

(c) Sketch the Fourier sine series of $f(t)$ for $t \in [-3, 3]$.

In each sketch, carefully mark the values of the Fourier series at discontinuities.

(a) 

(b) 

(c) 

Problem 4. (5 points)

(a) Suppose $y(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n$. How can we compute the a_n from $y(x)$? $a_n =$ [ ].

(b) Suppose $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi t}{3}\right) + b_n \sin\left(\frac{n\pi t}{3}\right) \right)$. How can we compute the a_n and b_n from $f(t)$?

$a_n =$ [ ] and $b_n =$ [ ].

(c) Determine the power series around $x = 0$: $e^{7x} =$ [ ]

(d) Determine the power series around $x = 0$: $\frac{1}{1 + x^2} =$ [ ]

Problem 5. (3 points) A mass-spring system is described by the equation $y'' + ky = \sum_{n=1}^{\infty} \frac{1}{n^2+7} \cos\left(\frac{nt}{4}\right)$.

For which values of k does resonance occur?

Problem 6. (3 points) Find a minimum value for the radius of convergence of a power series solution to

$$(x^2+1)y'' = \frac{y}{x+1} \quad \text{at } x = 2.$$

Problem 7. (4 points) Derive a recursive description of the power series for $y(x) = \frac{1}{1-3x+2x^2}$.

(extra scratch paper)