Midterm #1

MATH 332 — Differential Equations II
Friday, Feb 17, 2023

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 26 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (1+4+1 points) Consider the sequence a_n defined by $a_{n+2} = 2a_{n+1} + 3a_n$ and $a_0 = -2$, $a_1 = 6$.

(a) The next two terms are $a_2 = \boxed{}$ and $a_3 = \boxed{}$.

(b) A Binet-like formula for a_n is $a_n = \boxed{}$, and $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \boxed{}$.

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Problem 2. (3 points) Consider the following system of initial value problems:

$$\begin{aligned} y_1'' &= 3y_1' - 5y_2 \\ y_2'' &= y_1' - y_2' + 3y_1 \end{aligned} \qquad y_1(0) = -2,\ y_1'(0) = 1,\ y_2(0) = 0,\ y_2'(0) = 3$$

Write it as a first-order initial value problem in the form $\boldsymbol{y}' = M\boldsymbol{y}$, $\boldsymbol{y}(0) = \boldsymbol{c}$.

$M =$	$\boldsymbol{c} =$

Problem 3. (8 points) Fill in the blanks. None of the problems should require any computation!

(a) Consider a homogeneous linear differential equation with constant real coefficients which has order 4. Suppose $y(x) = 4x^2e^{-x} + 7$ is a solution. Write down the general solution.

(b) Determine a (homogeneous linear) recurrence equation satisfied by $a_n = (n+2)3^n - 7$.

You can use the operator N to write the recurrence. No need to simplify, any form is acceptable.

(c) If $e^{Mx} = \begin{bmatrix} 2e^{2x} - e^x & e^{2x} - e^x \\ -2e^{2x} + 2e^x & -e^{2x} + 2e^x \end{bmatrix}$, then $M^n =$ ____ .

(d) Let y_p be any solution to the inhomogeneous linear differential equation $y'' + 5y = 3e^{2x} + 5x$. Find a homogeneous linear differential equation which y_p solves.

You can use the operator D to write the DE. No need to simplify, any form is acceptable.

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Problem 4. (9 points) Let $M = \begin{bmatrix} 5 & 4 \\ 8 & 1 \end{bmatrix}$.

(a) Compute e^{Mx}.

(b) Solve the initial value problem $\boldsymbol{y}' = M\boldsymbol{y}$ with $\boldsymbol{y}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

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(extra scratch paper)