Midterm #1 – Practice

MATH 332 — Differential Equations II
Midterm: Friday, Feb 17, 2023

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches). Any typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Problem 1.

(a) Find the general solution to $y^{(5)} - 4y^{(4)} + 5y''' - 2y'' = 0$.

(b) Find the general solution to $y''' - y = e^x + 7$.

(c) Solve $y'' + 2y' + y = 2e^{2x} + e^{-x}$, $y(0) = -1$, $y'(0) = 2$.

(d) Find the general solution to $y'' - 4y' + 4y = 3e^{2x}$.

(e) Consider a homogeneous linear differential equation with constant real coefficients which has order 6. Suppose $y(x) = x^2 e^{2x} \cos(x)$ is a solution. Write down the general solution.

(f) Write down a homogeneous linear differential equation satisfied by $y(x) = 1 - 5x^2 e^{-2x}$.

(g) Let y_p be any solution to the inhomogeneous linear differential equation $y'' + xy = e^x$. Find a homogeneous linear differential equation which y_p solves. *Hint:* Do not attempt to solve the DE.

Problem 2.

(a) Write down a (homogeneous linear) recurrence equation satisfied by $a_n = 3^n - 2^n$.

(b) Write down a (homogeneous linear) recurrence equation satisfied by $a_n = n^2 3^n - 2^n$.

Problem 3. Consider the sequence a_n defined by $a_{n+2} = a_{n+1} + 6a_n$ and $a_0 = 3$, $a_1 = -1$.

(a) Determine the first few terms of the sequence.

(b) Find a Binet-like formula for a_n.

(c) Determine $\lim_{n \to \infty} \frac{a_{n+1}}{a_n}$.

Problem 4. Let $M = \begin{bmatrix} 1 & 4 \\ 6 & -1 \end{bmatrix}$.

(a) Determine the general solution to $\boldsymbol{a}_{n+1} = M\boldsymbol{a}_n$.

(b) Determine a fundamental matrix solution to $\boldsymbol{a}_{n+1} = M\boldsymbol{a}_n$.

(c) Compute M^n.

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Problem 5.

(a) Write the differential equation $y''' + 7y'' - 3y' + y = 0$ as a system of (first-order) differential equations.

(b) Consider the following system of initial value problems:

$$\begin{array}{l} y_1'' = 3y_1' + 2y_2' - 5y_1 \\ y_2'' = y_1' - y_2' + 3y_2 \end{array} \qquad y_1(0) = 1,\ y_1'(0) = -2,\ y_2(0) = 3,\ y_2'(0) = 0$$

Write it as a first-order initial value problem in the form $\boldsymbol{y}' = M\boldsymbol{y},\ \boldsymbol{y}(0) = \boldsymbol{y}_0$.

Problem 6. Let $M = \begin{bmatrix} 11 & -2 \\ 3 & 4 \end{bmatrix}$.

(a) Determine the general solution to $\boldsymbol{y}' = M\boldsymbol{y}$.

(b) Determine a fundamental matrix solution to $\boldsymbol{y}' = M\boldsymbol{y}$.

(c) Compute e^{Mx}.

(d) Solve the initial value problem $\boldsymbol{y}' = M\boldsymbol{y}$ with $\boldsymbol{y}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

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