Final Exam – Practice

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches). Any typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Problem 1. The final exam will be comprehensive, that is, it will cover the material of the whole semester.

  (a) Do the practice problems for both midterms.

  (b) Retake both midterm exams.

  (c) Do the problems below. (Solutions are posted.)

Problem 2. Find all eigenfunctions and eigenvalues of $y'' + \lambda y = 0$, $y(0) = 0$, $y(2) = 0$ in the case $\lambda > 0$.

Problem 3. Find all eigenfunctions and eigenvalues of

$$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y(3) = 0.$$

Problem 4. Find all eigenfunctions and eigenvalues of

$$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y'(3) = 0.$$

Problem 5. Find the solution $u(x, t)$, for $0 < x < 3$ and $t \geqslant 0$, to the heat conduction problem

$$u_t = 5u_{xx}, \quad u_x(0, t) = u_x(3, t) = 0, \quad u(x, 0) = 7 + 4\cos(\pi x).$$

Derive your solution using separation of variables (at some step you may refer to one of the EVP's above).

Problem 6. Find the solution $u(x, t)$, for $0 < x < 3$ and $t \geqslant 0$, to the heat conduction problem

$$2u_t = u_{xx}, \quad u_x(0, t) = 0, \ u(3, t) = 0, \quad u(x, 0) = 2\cos\left(\frac{\pi x}{2}\right) + 7\cos\left(\frac{3\pi x}{2}\right).$$

Derive your solution using separation of variables (at some step you may refer to one of the EVP's above).

Problem 7. For $t \geqslant 0$ and $x \in [0, 4]$, consider the heat flow problem:
$$\begin{aligned} u_t &= 2u_{xx} + e^{-x/2} \\ u_x(0, t) &= 3 \\ u(4, t) &= -2 \\ u(x, 0) &= f(x) \end{aligned}$$

Determine the steady-state solution and spell out equations characterizing the transient solution.

Problem 8. Find the solution $u(x, y)$, for $0 < x < 3$ and $0 < y < 2$, to

$$u_{xx} + u_{yy} = 0, \quad u(x, 0) = 0, \quad u(x, 2) = 0, \quad u(0, y) = \sin(\pi y) + 2\sin(7\pi y), \quad u(3, y) = 0.$$

Derive your solution using separation of variables.