Midterm #2

MATH 332 — Differential Equations II
Wednesday, Nov 13, 2024

Please print your name:

No notes, fancy calculators or tools of any kind are permitted. There are 31 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (7 points) Determine the equilibrium points of the system $\frac{\mathrm{d}x}{\mathrm{d}t} = (x+1)y$, $\frac{\mathrm{d}y}{\mathrm{d}t} = 2 - xy$ and classify their stability.

Problem 2. (3 points) Find a minimum value for the radius of convergence of a power series solution to

$$(x^2+4)y'' = \frac{y}{x+1} \quad \text{at } x = 2.$$

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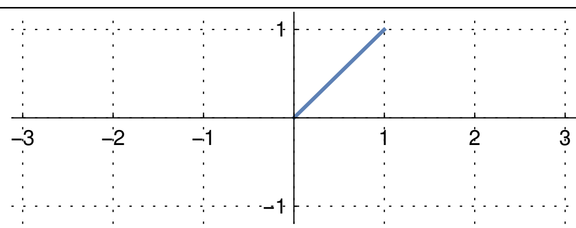
Problem 3. (6 points) Derive a recursive description of a power series solution $y(x)$ (around $x=0$) to the differential equation $y'' = 3x^2y' + 4y$.

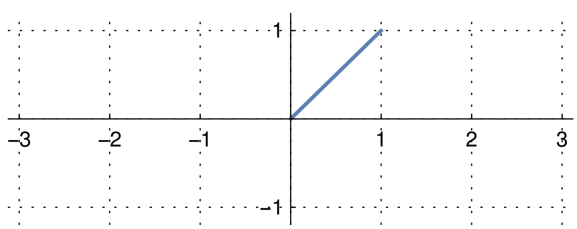
Problem 4. (4 points) Consider the function $f(t) = t$, defined for $t \in [0, 1]$. Sketch the following for $t \in [-3, 3]$.

(a) Fourier series of $f(t)$ (b) Fourier cosine series of $f(t)$ (c) Fourier sine series of $f(t)$

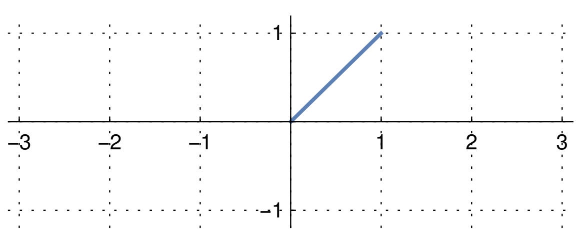
In each sketch, carefully mark the values of the Fourier series at discontinuities.



(a)



(b)



(c)

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Problem 5. (3 points) Let $y(x)$ be the unique solution to the IVP $y''=5+2(x-1)y^2$, $y(0)=1$, $y'(0)=2$. Determine the first several terms (up to x^3) in the power series of $y(x)$.

Problem 6. (3 points) A mass-spring system is described by the equation $my''+3y=\sum_{n=1}^{\infty}\frac{n}{n^2+1}\cos\left(\frac{nt}{5}\right)$.

For which values of m does resonance occur?

Problem 7. (5 points)

(a) Suppose $y(x)=\sum_{n=0}^{\infty}a_n(x+3)^n$. How can we compute the a_n from $y(x)$? $a_n=$ ______.

(b) Suppose $f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\left(\frac{n\pi t}{3}\right)+b_n\sin\left(\frac{n\pi t}{3}\right)\right)$. How can we compute the a_n and b_n from $f(t)$?

$a_n=$ ______ and $b_n=$ ______.

(c) Determine the power series around $x=0$: $3e^{-x}=$ ______

(d) Determine the power series around $x=0$: $\frac{3}{1+5x}=$ ______

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(extra scratch paper)